J. Math. Soc. Japan
Vol. 44, No. 3, 1992

# Blow-up sets and asymptotic behavior of interfaces for quasilinear degenerate parabolic equations in $\boldsymbol{R}^N$

Dedicated to Professor Takeshi Kotake on his 60th birthday

By Kiyoshi MOCHIZUKI and Ryuichi SUZUKI

(Received Nov. 6, 1990)
(Revised Aug. 13, 1991)

**Introduction.**

In this paper we shall consider the Cauchy problem

$$\partial_t\beta(u)=\Delta u+f(u) \quad \text{in} \quad (x,t)\in \boldsymbol{R}^N\times(0,T), \tag{0.1}$$

$$u(x,0)=u_0(x) \quad \text{in} \quad x\in \boldsymbol{R}^N, \tag{0.2}$$

where $\partial_t=\partial/\partial t$, $\Delta$ is the $N$-dimensional Laplacian and $\beta(v)$, $f(v)$ with $v\geqq 0$ and $u_0(x)$ are nonnegative functions.

Equation (0.1) describes the combustion process in a stationary medium, in which the thermal conductivity $\beta'(u)^{-1}$ and the volume heat source $f(u)$ are depending in a nonlinear way on the temperature $\beta(u)=\beta(u(x,t))$ of the medium.

Throughout this paper we assume

(A1) $\beta(v)$, $f(v)\in C^\infty(\boldsymbol{R}_+)\cap C(\bar{\boldsymbol{R}}_+)$, where $\boldsymbol{R}_+=(0,\infty)$ and $\bar{\boldsymbol{R}}_+=[0,\infty)$; $\beta(v)>0$, $\beta'(v)>0$, $\beta''(v)\leqq 0$ and $f(v)>0$ for $v>0$; $\lim_{v\to\infty}\beta(v)=\infty$; $f\circ\beta^{-1}$ is locally Lipschitz continuous in $[\beta(0),\infty)$.

(A2) $u_0(x)\geqq 0$, $\not\equiv 0$ and $\in B(\boldsymbol{R}^N)$ (bounded continuous in $\boldsymbol{R}^N$).

With these conditions the above Cauchy problem has a unique local solution $u(x,t)$ (in time) which satisfies (0.1) in $\boldsymbol{R}^N\times(0,T)$ in the following weak sense (see e.g., Oleinik et al. [**17**]), where $T>0$ is assumed sufficiently small.

DEFINITION 1. Let $G$ be a domain in $\boldsymbol{R}^N$. By a solution of equation (0.1) in $G\times(0,T)$ we mean a function $u(x,t)$ such that

1) $u(x,t)\geqq 0$ in $\bar{G}\times[0,T)$, and $\in B(\bar{G}\times[0,\tau])$ for each $0<\tau<T$.

2) For any bounded domain $\Omega\subset G$, $0<\tau<T$ and nonnegative $\varphi(x,t)\in C^2(\bar{\Omega}\times[0,T))$ which vanishes on the boundary $\partial\Omega$,

$$\int_\Omega\beta(u(x,\tau))\varphi(x,\tau)dx-\int_\Omega\beta(u(x,0))\varphi(x,0)dx$$
$$=\int_0^\tau\int_\Omega\{\beta(u)\partial_t\varphi+u\Delta\varphi+f(u)\varphi\}dxdt-\int_0^\tau\int_{\partial\Omega}u\partial_n\varphi dSdt, \tag{0.3}$$