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The Neumann and Dirichlet problems for elliptic operators

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1. Introduction.

Let D be a bounded C^1-domain in $\boldsymbol{R}^d$. In [3] E.B. Fabes, M. Jodeit JR. and N.M. Rivière proved that, for every $f \in L^p(\partial D)$ satisfying $\int f d\sigma = 0$, there exists a function u which is harmonic in D, and $\langle \nabla u(X), N_P \rangle$ converges to $f(P)$ with an exception of a set of surface measure zero as X tends to P nontangentially. The corresponding results have been obtained even for a Lipschitz domain D in the case $1 < p < 2 + \varepsilon$ (cf. [4], [2]).

On the other hand it is well-known that in $\boldsymbol{R}_+^{d+1}$ the Poisson integral of the Bessel potential $G_\alpha * f$ of each $f \in L^p(\boldsymbol{R}^d)$ converges not only nontangentially but also tangentially except for a set of an appropriately dimensional Hausdorff measure zero (cf. [1]).

In [7], for a bounded $C^{1,\alpha}$-domain D, we have studied the boundary behavior of the derivatives of solutions for the above Neumann problem, not up to an exception with a set of surface measure zero, but up to an exception with a set of β-dimensional Hausdorff measure zero for β satisfying $0 < \beta < d-1$.

In this paper we will consider the corresponding boundary behaviors of solutions of the Dirichlet and Neumann problems for uniformly elliptic differential operators.

Let L be a differential operator on $\boldsymbol{R}^d$ ($d \geqq 3$) defined by

$$L = \sum_{j,k=1}^d D_j(a_{jk} D_k), \tag{1.1}$$

where $D_j = \partial / \partial x_j$ and a_{jk} are of class $C^{1,\alpha}$ with $a_{jk} = a_{kj}$. Moreover L is assumed to be uniformly elliptic. This means that there exists a positive real number $\lambda > 1$ such that

$$\lambda^{-1} |\xi|^2 \leqq \sum_{j,k=1}^d a_{jk}(X) \xi_j \xi_k \leqq \lambda |\xi|^2$$

for all X, $\xi = (\xi_1, \cdots, \xi_d) \in \boldsymbol{R}^d$.

Let D be a bounded $C^{1,\alpha}$-domain in $\boldsymbol{R}^d$ and $0 < \beta < d-1$. To classify functions defined on ∂D, we use, as in [7], a countably sublinear functional γ_β and