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A measure on the space of smooth mappings and dynamical system theory

Dedicated to Professor Masahisa Adachi on his 60th birthday

By Masato TSUJII

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1. Introduction.

In order to get good descriptions of the properties of dynamical systems, we often exclude some set of systems which seem to have singular properties. In such cases, it is important whether we can ignore the excluded set of systems or not. For example, when we consider discrete smooth dynamical systems, we often neglect the systems which have non-hyperbolic periodic points, and the transversality theorem [3] says that such systems are exceptional. In fact, systems with non-hyperbolic periodic points form a countable union of stratified subsets of codimension one in the space of mappings in some sense [3]. But, in recent studies of dynamical system theory, we frequently consider complicated subsets such as Cantor sets in the space of mappings or in the parameter spaces of parameter families. (See [2], [4] and [5].) And there is no established idea to judge whether we can neglect such subsets or not. The argument that we can neglect the complements of open dense subsets (or residual sets) has been used in some cases and it was useful in the early stage of dynamical system theory. But recent developments show that this idea is not satisfactory one. Let us see the following examples.

EXAMPLE 1. Let $D^r(S^1)$ be the set of C^r-diffeomorphisms on the unit circle $S^1=\boldsymbol{R}/\boldsymbol{Z}$. Then it is easy to see that the set of diffeomorphisms with periodic points contains open dense subset in $D^r(S^1)$ for any $r\geqq 0$. But M. R. Herman showed in [4] that the Lebesgue measure of the set $\{t\in[0, 1]\,|\,f_t$ has no periodic point.$\}$ is positive for any C^1-family $\{f_t\}_{t\in[0,1]}$ in $D^3(S^1)$ with $\rho(f_0)\neq\rho(f_1)$ (ρ: rotation number).

EXAMPLE 2. In [5], S. Newhouse gave a result that the systems with infinitely many periodic sinks form residual set E in the parameter space for a certain class of one-parameter families of C^2-diffeomorphisms. This result implies that the set of systems with infinitely many periodic sinks contains a