J. Math. Soc. Japan
Vol. 44, No. 3, 1992

Remarks on metaplectic representations of $SL(2)$*

By Hiroyuki YOSHIDA

(Received June 7, 1991)

Introduction.

In a fundamental paper [9], Weil constructed oscillator representations of metaplectic groups. When specialized to the case $G=SL(2, k)$ where k is a local field whose characteristic is not 2, the construction gives a projective representation π of G realized on $L^2(k)$ such that

$$(\pi(\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix})F)(x) = \psi(bx^2)F(x), \tag{1}$$

$$(\pi(w)F)(x) = \gamma F^*(x), \tag{2}$$

for $F \in L^2(k)$. Here $w=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, ψ is a non-trivial additive character of k, F^* is the Fourier transformation of F with respect to ψ and γ is a constant independent of F. When lifted to the 2-fold covering group of G (if $k \neq \boldsymbol{C}$), π becomes an ordinary representation. An important problem, already suggested in [9], is to construct analogous representations for an n-fold covering group of G, $n \geqq 3$. A natural candidate is to replace x^2 by x^n in (1) and F^* by a suitably generalized Fourier transformation. This problem was solved by Kubota [3], [4] for $k=\boldsymbol{C}$ and by Yamazaki [10] for $k=\boldsymbol{R}$ and n is even.

In this paper, we shall give a conceptually simpler and unified treatment of these representations including the case $k=\boldsymbol{R}$, n is odd. We are going to sketch our idea intuitively. First observation is that we should start from a representation π (we choose it as a principal series representation corresponding to the parameter s, see the text) of an n-fold covering group $\tilde{G}$ of G and then should examine its Kirillov realization. Thus we realize π on a suitable (pre-Hilbert) space V of functions f on k such that the action of $\pi\,\widetilde{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}}$ is given by $f(x) \to \psi(bx)f(x)$. Here, for $g \in G$, $\tilde{g}$ denotes some naturally defined element of $\tilde{G}$ which projects to g (see § 1). Let $\tilde{V}$ be the vector space of all functions F on k defined by $F(x)=f(x^n)$, $x \in k$, $f \in V$. Set $F=\iota(f)$ and put

$$\tilde{\pi}(g)F = \iota(\pi(g)f) \qquad \text{for } g \in \tilde{G}\,. \tag{3}$$

---

* This work was partially supported by the Inamori Foundation.