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Homogenization of cadlag processes

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1. Introduction.

Let L be a d-dimensional Lévy type operator:

$$Lf(x)=\frac{1}{2}\sum_{i,j=1}^{d}a_{ij}(x)\partial_{x_i}\partial_{x_j}f(x)+\sum_{i=1}^{d}b_i(x)\partial_{x_i}f(x)$$
$$+\int_{R^d}\Big\{f(x+y)-f(x)-\sum_{i=1}^{d}y_i\partial_{x_i}f(x)\Big\}\nu(x,dy), \tag{1.1}$$

where $\partial_{x_i}=\partial/\partial x_i$, $a(x)=(a_{ij}(x))$ is a nonnegative definite symmetric $d\times d$ matrix, $b(x)=(b_i(x))$ is a d-vector, and $\nu(x, dy)$ is a Lévy measure on $\boldsymbol{R}^d$ for each $x\in\boldsymbol{R}^d$: $\nu(x,\{0\})=0$ and $\int_{R^d}|y|^2/(1+|y|^2)\nu(x,dy)<\infty$, $x\in\boldsymbol{R}^d$. Denote by $\{X^L(t)\}$ a cadlag process on $\boldsymbol{R}^d$ governed by L. Here a cadlag process means a Markov process whose sample paths are right continuous and have left hand limits. In this paper we will consider a homogenization problem associated with $\{X^L(t)\}$. Namely, under the condition of periodicity of $a(x)$, $b(x)$ and $\nu(x, dy)$ in x and some additional condition, we will study to what process the scaled process $\{\varepsilon X^L(t/\varphi(\varepsilon))\}$ converges as $\varepsilon\downarrow 0$ with some suitable scaling function φ.

Horie, Inuzuka and Tanaka [3] has already investigated the same problem in the case where $d=1$, $a(x)\equiv 0$ and Lévy measure is absolutely continuous with respect to the Lebesgue measure. More precisely, let

$$Af(x)=b(x)f'(x)+\int_{-\infty}^{\infty}\{f(x+y)-f(x)-yf'(x)\}c(x,y)n(y)dy, \tag{1.2}$$

where $b(x)$ and $c(x, y)$ are periodic in x with period 1 and c is strictly positive, and $n(y)=\gamma_-|y|^{-1-\alpha_0}$ $(y<0)$, $=\gamma_+y^{-1-\alpha_0}$ $(y>0)$, for some $\alpha_0\in(1,2)$ and nonnegative numbers γ_-, γ_+ with $\gamma_-+\gamma_+>0$. If there exist the limits $c_\pm=\lim_{r\to\pm\infty}(1/r)\int_0^r dy\int_T c(x,y)\mu(dx)$, μ being the invariant probability measure of the cadlag process $\{\mathfrak{X}^A(t)\}$ on $\boldsymbol{T}\equiv\boldsymbol{R}/\boldsymbol{Z}$ induced by $\{X^A(t)\}$, then the scaled cadlag process $\{\varepsilon X^A(t/\varepsilon^{\alpha_0})\}$ converges to a stable process $\{X^{A^*}(t)\}$ in law as $\varepsilon\downarrow 0$. The generator A^* of the process $\{X^{A^*}(t)\}$ is given by

$$A^*f(x)=\int_{-\infty}^{\infty}\{f(x+y)-f(x)-yf'(x)\}c^*(y)n(y)dy, \tag{1.3}$$