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The maximum Markovian self-adjoint extensions of generalized Schrödinger operators

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0. Introduction.

Let G be an open set in $\boldsymbol{R}^d$ and let m be a Radon measure on G. Let S be a symmetric linear operator on $L^2(G, m)$ with the domain $\mathcal{D}[S]$ being dense in $L^2(G, m)$. Let us define a symmetric form by $\mathcal{E}_{(S)}(u, v)=(-Su, v)_m$, $u, v\in \mathcal{D}[S]$ and assume that the symmetric form $\mathcal{E}_{(S)}$ is Markovian in the sense of [8]. Then, the Friedrichs extension of S, the self-adjoint operator associated with the smallest closed extension of $\mathcal{E}_{(S)}$, generates Markovian semigroup ([8; Theorem 2.11]). Let us denote by $\mathcal{A}_{\mathcal{M}}(S)$ the family of all self-adjoint extensions which generate Markovian semigroups, and let us call an element of $\mathcal{A}_{\mathcal{M}}(S)$ a *Markovian extension* of S. Recall that semi-order "$<$" on $\mathcal{A}_{\mathcal{M}}(S)$ is defined by

$$A_1 < A_2 \quad \text{if } \mathcal{D}[A_1]\subset\mathcal{D}[A_2] \text{ and}$$
$$(\sqrt{-A_1}u, \sqrt{-A_1}u)_m \geqq (\sqrt{-A_2}u, \sqrt{-A_2}u)_m \qquad \text{for } u\in\mathcal{D}[\sqrt{-A_1}].$$

Then, the Friedrichs extension of S is the minimum one of $\mathcal{A}_{\mathcal{M}}(S)$ with this semi-order. Now, it is natural to ask whether the maximum element of $\mathcal{A}_{\mathcal{M}}(S)$ exists and what is the maximum one if it exists.

In the case that m is the Lebesgue measure and S is the Laplacian Δ defined on $C_0^\infty(G)$ (in notation $\Delta\upharpoonright C_0^\infty(G)$), the maximum element of $\mathcal{A}_{\mathcal{M}}(\Delta)$ is the self-adjoint operator associated with the Sobolev space $W^{1,2}(G)$ ([8; Theorem 2.3.1]). Here $C_0^\infty(G)$ is the space of infinitely differentiable functions with compact support in G. In this paper, we shall extend this result to "*generalized Schrödinger operators*". More precisely, let ρ be a measurable function on G which is strictly positive almost everywhere and locally square integrable with respect to the Lebesgue measure λ^d. Let us assume that ρ is differentiable in the sense of the Schwartz distribution and its derivatives $\nabla_i\rho$ are also locally square integrable. Then, we define a generalized Schrödinger operator by

$$L_\rho\varphi=\Delta\varphi+2\sum_{i=1}^{d}\nabla_i\rho/\rho\cdot\nabla_i\varphi, \qquad \varphi\in C_0^\infty(G), \tag{0.1}$$