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Non-commutative lens spaces

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0. Introduction.

It is based on the classical Gelfand-Naimark theorem for commutative C^*-algebras to think of the theory of C^*-algebras as non-commutative general topology. In the early part of the 1980's, the explosive development of K-theory for C^*-algebras began to make a much stronger case for C^*-algebras as non-commutative topology ([**BDF**], [**Co1**], [**El1**], [**Ka**], [**PV**], [**Ro**], interalia). As part of such a development of K-theory in operator algebras, Connes began to investigate the theory of non-commutative differential geometry in [**Co2**]. Since then, many operator algebraists have investigated non-commutative geometry and topology in the setting of C^*-algebras, for example, higher dimensional non-commutative tori in [**El2**], Yang-Mills on non-commutative 2-tori in [**CR**], the Künneth and universal coefficient theorems of K-theory in [**RS**], deformation quantization by Poisson structures in [**Ri2**], and quantum groups in [**Wo1**], [**Wo2**]. These special non-commutative C^*-algebras, resembling ordinary manifolds in a sense, are sometimes called "non-commutative manifolds". Although we have not found yet the decisive definition of non-commutative manifold, these C^*-algebras are often considered as non-commutative manifolds, and the theory of non-commutative geometry and topology on them has been developing. Among the examples of non-commutative manifolds, the irrational rotation C^*-algebras, called non-commutative tori, have been playing a central rôle, especially in Connes's non-commutative differential geometry. Since many examples support the prosperity of ordinary topology and differential geometry today, the presentation of new examples of "non-commutative manifolds" would perhaps be significant for the development of the theory of non-commutative manifolds in the future. With this in mind, we shall provide some new examples of non-commutative manifolds in the category of C^*-algebras and investigate their structures.

This paper is the second step (following [**Ma1**]) of our plan, in which we aim toward the construction of a theory of non-commutative 3-manifolds.

The first author (in [**Ma1**]) has introduced one method of deforming a 3-sphere into non-commutative C^*-algebras which gives rise to a one-parameter