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Three contributions to the homotopy theory of the exceptional Lie groups G_2 and F_4*

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1. Statement of results.

In this paper, we prove three theorems related to the homotopy theory of the exceptional Lie groups G_2 and F_4. These results will be useful in work of the first author with Bendersky and Mimura, which seeks to calculate v_1-periodic homotopy groups of all exceptional Lie groups.

Our first result, which will be proved in Section 2, should be useful in determining the homotopy groups of the homogeneous space F_4/G_2, and consequently in deducing information about $\pi_*(F_4)_{(2)}$ from information about $\pi_*(G_2)_{(2)}$.

THEOREM 1.1. *There is a 2-local fibration*

$$S^{15} \longrightarrow F_4/G_2 \longrightarrow S^{23}.$$

Such a fibration is known to exist localized at primes $\geqq 5$, ([**21**]) and to not exist at the prime 3. ([**7**])

Our second result is relevant to F_4 because of the equivalence $F_4/Spin(9) = \Pi$, where Π denotes the Cayley projective plane ([**6**]).

THEOREM 1.2. *There is a fibration*

$$S^7 \longrightarrow \Omega\Pi \longrightarrow \Omega S^{23}.$$

This result, which will be proved in Section 3, might allow one to extend the range of calculation of $\pi_*(\Pi)$ begun in [**20**]. In particular, it implies both upper- and lower-bounds for p-exponents of Π, which are defined by

$$\exp_p(\Pi) = \max\{e : \pi_*(\Pi) \text{ has an elements of order } p^e\}.$$

If $p \geqq 5$, then it is known (e. g., [**20**]) that the fibration of our Theorem 1.2 exists as a product, and so $\exp_p(\Pi) = \exp_p(S^{23}) = 11$, by [**10**]. Our theorem implies that

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