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On special values of Selberg type zeta functions on $SU(1, q+1)$

By Koichi TAKASE

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§ 0. Introduction.

There is mystery in the arithmetic nature of the special values of $\zeta(s)=\sum_{n=1}^{\infty}n^{-s}$ at the odd integers greater than one.

It is widely believed that the special values of the Dedekind zeta function $\zeta_K(s)$ of an algebraic number field K at the positive integer m is written in the form

$$\zeta_K(m)=R\cdot P\cdot A$$

where $R=\mathrm{vol}(\Gamma\backslash \boldsymbol{R}^r)$ is the (higher) regulator with $r=\mathrm{ord}_{s=1-m}\zeta_K(s)$ and $\Gamma\subset \boldsymbol{R}^r$ a $\boldsymbol{Z}$-lattice, P is the period and A is an algebraic number called the algebraic part of the special value $\zeta_K(m)$. A typical example is the residue formula at $s=1$, that is, $\zeta_K(s)$ has a simple pole at $s=1$ and

$$\operatorname*{Res}_{s=1}\zeta_K(s)=R(K)\cdot P\cdot A$$

where $R(K)=\mathrm{vol}(U_K\backslash \boldsymbol{R}^{r_1+r_2-1})$ is the usual regulator of K with $r_1+r_2-1=\mathrm{ord}_{s=0}\zeta_K(s)$, $P=2^{r_1}(2\pi)^{r_2}$, and $A=h/(w\sqrt{|D|})$. Here U_K is the unit group of the maximal order of K, r_1 (resp. r_2) is the number of the real (resp. complex) places of K, h is the class number of K, w is the number of the roots of unity contained in K, and D is the absolute discriminant of K.

In this paper, we will show that special values of Selberg zeta functions are also written as a product of "regulator" and "period".

In § 1, we will recall basic facts on the irreducible unitary representations of the special unitary group $SU(1, q+1)$ of signature $(1, q+1)$ $(q>0)$. The unitary dual of a real rank one semi-simple Lie group is determined by [**BSB**]. We will recall a result of Kraljevic [**Kr**] in which we can find a parametrization of the irreducible unitary representations of $SU(1, q+1)$ and the irreducible decomposition of them restricted to a maximal compact subgroup K of $SU(1, q+1)$. We will give a connection between the Harish-Chandra parametrization of square-integrable representations of $SU(1, q+1)$ and the parametrization of Kraljevic.