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The Alder-Wainwright effect for stationary processes with reflection positivity

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1. Introduction.

The aim of this paper is to refine the results of Okabe [**13**] on the Alder-Wainwright effect for KMO-Langevin equations. Here by the *Alder-Wainwright effect,* we mean the long-time tail behaviors ($\propto t^{-(p+1)}$, $p>0$) of autocorrelation functions for stationary processes of non-Markovian type. This long-time tail was first observed by Alder and Wainwright in their computer experiment of molecular dynamics ([**1**] and [**2**]). Their numerical calculation suggested that the slowly developing viscous flow around a particle could explain the long-time tail behaviors.

The usual Langevin equation of Ornstein-Uhlenbeck type is not adequate for this Brownian particle because its autocorrelation function decays exponentially. This equation neglects the effect of the fluid flow around the particle which is generated by the accelerated motion of the particle. The appropriate hydrodynamic drag force on a spherical particle moving arbitrarily in $\boldsymbol{R}^3$ has been calculated by Stokes and Boussinesq by solving the linearlized Navier-Stokes equation ([**7**]). Then, the Langevin equation with this drag force becomes

$$m^*\frac{dX(t)}{dt} = -6\pi r\eta X(t)-6\pi r^2\left(\frac{\rho\eta}{\pi}\right)^{1/2}\int_{-\infty}^{t}\frac{1}{\sqrt{t-s}}\frac{dX(s)}{ds}ds+W(t), \tag{1.1}$$

where m^* is an effective mass given by $m^*=m+(2/3)\pi r^3\rho$. Here we consider the motion of a sphere of radius r and mass m moving with an arbitrary velocity $X(t)$ at time t in a fluid with viscosity η and density ρ subject to a random force $W(t)$ at time t. The second term of the right-hand side of (1.1) corresponds to the effect of the accelerated fluid flow around the particle. It has been shown (e.g. [**6**] and [**15**]) that the correlation function $R(t)$ of the stationary solution X of (1.1) has a long-time tail $\propto t^{-3/2}$ as $t\to\infty$, which agrees with the above experiment. We remark that now the Alder-Wainwright effect has been observed by not only a computer experiment but also a physical experiment ([**14**]).