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The Gorensteinness of symbolic Rees algebras for space curves

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1. Introduction.

Let A be a regular local ring and $\mathfrak{p}$ a prime ideal in A. We put $R_s(\mathfrak{p})=\sum_{n\geq 0}\mathfrak{p}^{(n)}t_n$ (here t denotes an indeterminate over A) and call it the symbolic Rees algebra of $\mathfrak{p}$. Our purpose is to discuss when $R_s(\mathfrak{p})$ is a Gorenstein ring.

The problem when $R_s(\mathfrak{p})$ is Noetherian is, of course, more fundamental and has been studied by many authors from several points of view (cf. [**3**, **4**, **5**, **10**, **11**, **12**, **13**, **14**, **16**, **19**, **20**, **21** and **22**]). The finite generation problem on the A-algebra $R_s(\mathfrak{p})$ was raised by R. C. Cowsik [**3**], showing that $\mathfrak{p}$ is a set-theoretic complete intersection in A if $\dim A/\mathfrak{p}=1$ and if $R_s(\mathfrak{p})$ is a finitely generated A-algebra. If A is not regular, there is no hope in general of $R_s(\mathfrak{p})$ being Noetherian (cf. e.g., [**5**, Sect. 5]), as was firstly noticed by D. Rees [**19**] in his construction of a counterexample to the Zariski problem. Nevertheless even though A is regular the rings $R_s(\mathfrak{p})$ are not necessarily Noetherian. P. Roberts [**20**] gave such examples, passing to Nagata's counterexamples [**18**] to the 14-th problem of Hilbert. And as far as we know, Cowsik's problem seems still open for general prime ideals $\mathfrak{p}=\mathfrak{p}(n_1, n_2, n_3)$ of $A=k[[X, Y, Z]]$ (the formal power series ring over a field k) defining space monomial curves $X=t^{n_1}$, $Y=t^{n_2}$ and $Z=t^{n_3}$ with $GCD(n_1, n_2, n_3)=1$.

We look now at a prime ideal $\mathfrak{p}$ of height 2 in a 3-dimensional regular local ring A with maximal ideal $\mathfrak{m}$. Then in his remarkable paper [**12**] C. Huneke gave the following criterion for $R_s(\mathfrak{p})$ to be a Noetherian ring:

(*) *If there exist $f\in\mathfrak{p}^{(k)}$ and $g\in\mathfrak{p}^{(l)}$ with positive integers k, l such that $\mathrm{length}_A(A/(f, g, x)A)=kl\cdot\mathrm{length}_A(A/\mathfrak{p}+xA)$ for some $x\in\mathfrak{m}\setminus\mathfrak{p}$, then $R_s(\mathfrak{p})$ is Noetherian. When the field $A/\mathfrak{m}$ is infinite, the converse is also true.*

With this criterion Huneke explored prime ideals $\mathfrak{p}=\mathfrak{p}(n_1, n_2, n_3)$ and guaranteed that $R_s(\mathfrak{p})$ is Noetherian, if $\min(n_1, n_2, n_3)=4$.

In the present paper we would like to succeed Huneke's research, mainly asking for similar practical criteria as his for $R_s(\mathfrak{p})$ to be Gorenstein. It might

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