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Extremal almost periodic states on C^*-algebras

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1. Introduction.

Let (A, G, α) be a C^*-dynamical system, namely, a triple consisting of a C^*-algebra A, a locally compact group G and a group homomorphism α from G into the automorphism group of A such that $G \ni t \to \alpha_t(x)$ is continuous for each x in A. Assume that A is unital for a while (it will be irrelevant in Section 2 whether or not a C^*-algebra possesses the identity). Then the state space of A is weakly* compact. In decomposition theory of states (cf. [**1**, 4.1-4.4]), we are interested in decomposing a given state as a convex combination of states which are extremal points of some closed convex subset of the state space endowed with the weak* topology. The closed convex subset might be given directly by some physical requirement. In the covariant situation, usually the set of α-invariant states is considered as such a closed convex subset. Extremal points in the set of α-invariant states are called ergodic states (or α-ergodic states), and some of their characterizations are given in [**1**, Theorems 4.3.17 and 4.3.20].

Now assume that G is a locally compact *abelian* group. Recall that a state φ of A is called an *almost periodic state* if, for each x in A, the function $G \ni t \to \varphi(\alpha_t(x))$ is the uniform limit of a family of finite linear combinations of characters of G. Then we turn our attention to considering the decomposition of a given state into the weak* closure of almost periodic states (cf. [**1**], [**2**]). Here note that every α-invariant state is automatically almost periodic. At the first stage in this paper, we shall examine conditions under which an α-ergodic state becomes an extremal point in the weak* closure of almost periodic states. When an α-ergodic state becomes an extremal point in the weak* closure of almost periodic states, such a state shall be named an *ergodic state of almost periodic type*, together with the explicit definition, in Section 2. We shall consider also the class of states corresponding to centrally ergodic states (see [**1**, § 4.3.2] for the definition of a centrally ergodic state), and every state belonging to such a class shall be called a *centrally ergodic state of almost periodic type*, whose explicit definition shall be given later. In the latter half of this

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