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A general method of axiomatizing fragments

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The purpose of this paper is to present a general method of axiomatizing fragments of first order theories. Smorynski [**6**] presented a *new semi-model-theoretic* method of axiomatizing fragments. Our method in this paper is *proof-theoretic*. For the sake of contrast, we prove several results in [**6**] by our method. The method in this paper is a generalization of one in [**9**], and a development of one in [**3**] and [**4**]. Motohashi [**4**] introduced approximation theory of uniqueness conditions by existence conditions, and gave an axiomatization theorem for intuitionistic theories with equality which are axiomatized by uniqueness conditions and existence conditions. We modify the notion of approximations in [**4**], and give an axiomatization theorem for any intuitionistic theories with equality. We introduce the notion of *inference forms*, which plays an important role in our discussion. Inference forms are figures expressing inference rules such that first order logistics are formalized by sets of inference forms. For a given fragment of a theory, by choosing a suitable set of inference forms which formalizes the theory, we can construct a series of axioms which axiomatizes the fragment.

§1 is a preliminary section in which we introduce several basic notations and notions. In §2, we introduce the notion of inference forms, and give a cut elimination theorem for logistics formalized by sets of inference forms. The proofs in the following sections are based on the cut elimination theorem. In §3, first, we introduce the notion of approximations which is a modification of one in [**4**]. Then, we give several theorems for axiomatizing fragments by approximations. In §4, we prove several axiomatization results, some of which are proved in [**6**]. §5 is preparatory to axiomatization theorems in §6. In §6, we give an axiomatization theorem for classical theories with equality and an axiomatization theorem for intuitionistic theories with equality.

§1. Preliminary.

We are concerned with Gentzen-type systems: A *logistic* (or a *theory*) consists of initial sequents and inference figures. In *intuitionistic* logistics, the suc-