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On the mod p cohomology of the spaces of free loops on the Grassman and Stiefel manifolds

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§ 0. Introduction.

Let ΩX be a space of loops on X and ΛX a space of free loops on X. We will call a fibration $\Omega X \hookrightarrow \Lambda X \xrightarrow{\pi} X$ a free loop fibration on X where $\pi(w)=w(1)$ for $w \in \Lambda X$. Let $\boldsymbol{K}_p$ be a field of characteristic p. When is ΩX totally non-homologous to zero in ΛX with respect to a field $\boldsymbol{K}_p$?

We call a commutative algebra $\Lambda(y_1, \cdots, y_l) \otimes \boldsymbol{K}_p[x_1, \cdots, x_n]/(\rho_1, \cdots, \rho_m)$ over a field $\boldsymbol{K}_p$ is *GCI* algebra if $\rho_1, \cdots, \rho_m$ is a regular sequence (see [**4**; p. 95]) or $m=0$ where deg y_i is odd and deg x_i is even if $p \neq 2$. (see [**5**; Definition, p. 893].) In [**5**], L. Smith has proved the following.

THEOREM 1 ([**5**; Theorem 4.1]). *Let X be a simply connected space such that $H^*(X; \boldsymbol{K}_0)$ is a GCI algebra. Then ΩX is totally non-homologous to zero in ΛX with respect to $\boldsymbol{K}_0$ if and only if $H^*(X; \boldsymbol{K}_0)$ is a free commutative algebra, in which case $H^*(\Lambda X; \boldsymbol{K}_0) \cong H^*(X; \boldsymbol{K}_0) \otimes H^*(\Omega X; \boldsymbol{K}_0)$ as an algebra.*

In this paper, using methods which L. Smith has given in [**5**], we will examine whether ΩX is totally non-homologous to zero in ΛX with respect to $\boldsymbol{K}_p$ for cases where $X=U(m+n)/U(m)\times U(n)$, $Sp(m+n)/Sp(m)\times Sp(n)$, $Sp(n)/U(n)$, $SO(m+n)/SO(n)$, $SU(m+n)/SU(n)$, $Sp(m+n)/Sp(n)$, $\mathfrak{C}P(2)$ and $p \geqq 2$.

In order to obtain our results, we will consider the Eilenberg-Moore spectral sequence of a fibre square

$$\mathcal{F}(X) := \begin{array}{ccc} \Lambda X & \longrightarrow & X \\ \downarrow & & \downarrow \Delta \\ X & \underset{\Delta}{\longrightarrow} & X \times X \end{array} \qquad \text{(see [5])},$$

where Δ is a diagonal map. Throughout this paper, $\mathcal{F}(X)$ means the above fibre square.

For a space X, let $T(X)$ denote a set of prime numbers p such that ΩX is totally non-homologous to zero in ΛX with respect to $\boldsymbol{K}_p$.