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On microhyperbolic mixed problems

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Introduction.

Let us consider the following Dirichlet problem on $\Omega=\{t\in\boldsymbol{R};\ t>0\}\times\boldsymbol{R}_x^n$.

$$
(0.1)\qquad \begin{cases} P\,u(t,x)=f(t,x) & \text{in } \Omega, \\ u(+0,x)=g(x) & \text{on } \partial\Omega. \end{cases}
$$

Here $\partial\Omega=\{t=0\}\times\boldsymbol{R}_x^n$, and P is an analytic differential operator of order 2 defined on $\bar{\Omega}=\Omega\cup\partial\Omega$ of the form

$$
(0.2)\qquad P=D_t^2+A_1(t,x,D_x)D_t+A_2(t,x,D_x)
$$

with $D_t=\partial/\partial t$, $D_x=(\partial/\partial x_1,\cdots,\partial/\partial x_n)$. We study the problem (0.1) in the space of hyperfunctions, and thus $f(t,x)$ and $g(x)$ are hyperfunctions defined on Ω and $\partial\Omega$ respectively. Moreover we assume that $f(t,x)$ is mild on $t=+0$. This means that $f(t,x)$ belongs to a class of hyperfunctions for which the boundary values $D_t^jf(+0,x)$ $(j=0,1,2,\cdots)$ to $\partial\Omega$ are well-defined (see §1). Under this assumption, it follows from the non-charactericity of $\partial\Omega$ that every solution on Ω to the first equation of (0.1) becomes mild, and in particular that the second equation of (0.1) makes sense.

Let $u(t,x)$ be a hyperfunction solution to (0.1) in Ω. Then taking the canonical extensions $\tilde{u}(t,x)$ and $\tilde{f}(t,x)$ of $u(t,x)$ and $f(t,x)$ respectively, we get the identities

$$
(0.3)\qquad P\tilde{u}(t,x)=\tilde{f}(t,x)+g(x)\delta'(t)+(D_tu(+0,x)+A_1(0,x,D_x)g(x))\cdot\delta(t)
$$

and

$$
(0.4)\qquad tP\tilde{u}(t,x)=t\tilde{f}(t,x)-g(x)\delta(t)\quad \text{in } \boldsymbol{R}_t\times\boldsymbol{R}_x^n.
$$

Here the correspondence $u\to\tilde{u}$ is a well-defined operation on mild hyperfunctions, which is similar to the cut-off operation by the Heaviside function $Y(t)$.

Conversely, it is easy to see that every hyperfunction solution to (0.4) with condition $\operatorname{supp}\tilde{u}\subset\{t\geqq 0\}$ gives a solution to (0.1). Thus we can reduce the Dirichlet problem (0.1) to studying the local or global cohomology groups of the complex of sheaves