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On the Kervaire classes of homotopy real projective spaces

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The purpose of this paper is to study the mutual dependence of Kervaire classes for normal maps of a real projective space P^n.

A smooth free involution T on a homotopy sphere Σ^n defines a homotopy equivalence $\varphi: \Sigma^n/T \to P^n$ whose normal invariant is denoted by $\nu(\varphi) \in [P^n, F/O]$. By restricting $\nu(\varphi)$ to a subspace $P^m \subset P^n$, we obtain a surgery obstruction that lies in Wall's surgery obstruction group $L_m(\boldsymbol{Z}/2, (-1)^{m+1})$ which is isomorphic to $\boldsymbol{Z}/2$ unless $m \equiv 1 \bmod 4$ ([**12**], 13A). Suppose that M^m is an even dimensional smooth manifold and let $f \in [M, F/O]$ be a normal map. Then the Kervaire obstruction for f is given by the Sullivan's characteristic variety formula ([**10**]) as follows:

$$c(f) = \langle V(M)^2 \sum_i f^* H^* K_{2i}\,,\ [M] \rangle\,,$$

where $V(M)$ is the total Wu class of M^m, $[M]$ is the mod 2 fundamental homology class in $H^m(M^m, \boldsymbol{Z}/2)$, $K_{2i} \in H^{2i}(F/TOP, \boldsymbol{Z}/2)$ is the Sullivan-Kervaire class and $H: F/O \to F/TOP$ is the natural map.

When m is even, the formula above enables us to write down the surgery obstruction for $\nu(\varphi)|P^m$ in terms of the Kervaire classes of $\nu(\varphi)$. Giffen, in his works on Brieskorn involutions on homotopy spheres bounding parallelizable manifolds ([**4**], [**5**]), showed that in these examples all the Kervaire classes in different degrees (up to the dimension of the manifold) are either all zero or all nonzero. So we may ask if simultaneous vanishing or non-vanishing of the Kervaire classes occurs for arbitrary free involutions on a homotopy n-sphere Σ^n that bound a parallelizable $(n+1)$-manifold when $n \equiv 1 \bmod 4$.

Another motivation for the present work comes from the problem of Dovermann, Masuda and Schultz ([**3**], 4.12). They ask for a reasonable estimate of $M(q)$ such that the restriction map

$$[CP^m, \mathrm{Cok}\, J_{(2)}] \longrightarrow [CP^q, \mathrm{Cok}\, J_{(2)}]$$

is trivial for all $m \geqq M(q)$. Actually they proved

THEOREM ([**3**], 4.8). *Let $i+1=2^N$. Then there exists an M such that the*