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Global Sebastiani-Thom theorem for polynomial maps

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1. Introduction.

There are several methods for studying the topological type of affine hypersurfaces. Kouchnirenko and M. Oka have investigated in [**Ko**], [**O2**], [**O3**], [**O4**] the relation between Newton boundary and the topology of the generic fiber, Hà and Lê [**H-L**] determined the bifurcation set of polynomial maps with two variables using the Euler characteristic of the fibers. Another approach is due to Broughton [**Br1**], [**Br2**] who have introduced and studied the class of "tame" polynomials. His results have been extended by the author for the larger class of "quasitame" polynomials [**Ne**].

In this note we establish a Sebastiani-Thom type result. More precisely: Let $g: \boldsymbol{C}^n \to \boldsymbol{C}$ and $h: \boldsymbol{C}^m \to \boldsymbol{C}$ be polynomial maps with bifurcation sets Λ_g resp. Λ_h. We consider the sum-map $f: \boldsymbol{C}^n \times \boldsymbol{C}^m \to \boldsymbol{C}$, $f(x, y) = g(x) + h(y)$. We prove the following

THEOREM.

a) *The bifurcation set of f is contained in $\Lambda_g + \Lambda_h$.*

b) *The generic fiber of f is homotopic equivalent with the join space of the generic fibers of the polynomial maps g and h.*

c) *The global algebraic monodromy of f (around all the bifurcation points) is induced by the join of the global geometric monodromies of g and h. (In particular it can be determined in terms of the global algebraic monodromies of g and h).*

This result extends the results of Sebastiani-Thom [**Se-T**] and K. Sakamoto [**Sa1**], [**Sa2**] (in the local case) and M. Oka [**O1**] in the special case of weighted homogeneous polynomials. The proof is based on a new technique which applies to the general (global) case of the polynomials (without $\boldsymbol{C}^*$-action).

We are indebted to the referee for suggesting us the proof of Theorem 3.2 which is more natural and simple than our original proof.