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Strongly nonmultidimensional theories

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0. Introduction.

It is well-known that a countable theory is $\aleph_1$-categorical if and only if it is ω-stable and unidimensional. Tsuboi [7] has shown that a countable stable theory is almost strongly minimal if and only if it can be extended to a strongly unidimensional theory by adding finitely many constants, and he studied the notion of strongly two-dimensional theories in [8] and obtained some nice structure theorems for the big model of a theory with this property.

In the present paper, we define and study the notion of the strongly κ-dimensional theories. Our results extends many of the results in Tsuboi [8]. A stable theory T is called *strongly κ-dimensional* if all the types of T (with parameters in the big model) can be classified into κ classes such that any two types in the same class are not almost orthogonal. T is called *strongly non-multidimensional* if it is strongly κ-dimensional for some cardinal κ. We show that a strongly κ-dimensional theory is superstable and a strongly ω-dimensional countable theory is ω-stable. This seems to be significant since there are non-superstable two-dimensional theories and countable non-ω-stable unidimensional theories.

Since a strongly κ-dimensional theory T is superstable and nonmultidimensional, choosing a maximal orthogonal set of regular types on an a-model, every non-algebraic type is not orthogonal to one of the members of this set. The cardinality of this set is called the dimensionality of T and denoted $\mu(T)$. We define the strong dimensionality of T to be the smallest cardinal κ such that T is strongly κ-dimensional. If every non-algebraic type is not almost orthogonal to some member of the given set of regular types, then it is easy to see that the theory is strongly nonmultidimensional. We show that a theory can be extended to a strongly nonmultidimensional theory by adding constants if and only if it can be extended to a theory such that every non-algebraic type is not almost orthogonal to some member of a given maximal orthogonal set of regular types on an a-model by adding constants. Therefore we have that the strong dimensionality coincides with the usual dimensionality. For a countable

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