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Indivisibility of class numbers of totally imaginary quadratic extensions and their Iwasawa invariants

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§ 0. Introduction.

We denote by l an odd prime number. Hartung [2] proved that there exist infinitely many imaginary quadratic fields whose class numbers are not divisible by l. In this paper, we generalize this result to the case of totally imaginary quadratic extensions over a totally real algebraic number field. Moreover we generalize the result due to Horie [3] on Iwasawa invariants of basic $\boldsymbol{Z}_l$-extensions.

We denote by F a totally real algebraic number field and by m its degree over the field $\boldsymbol{Q}$ of rational numbers. We denote by $n(p)$ for a prime p the maximum value of n such that the primitive p^n-th roots ζ_{p^n} of unity are at most of degree 2 over F. If F is fixed we have $n(p)=0$ for almost all p. So we put $w_F=2^{n(2)+1}\prod_{p\neq 2}p^{n(p)}$. We denote by $\zeta_F(s)$ the Dedekind zeta function of F. We know by Serre [9] that $w_F\zeta_F(-1)$ is a rational integer. We denote by h_K the class number of an algebraic number field K. The relative class number $h_{K/F}=h_K/h_F$ is an integer when K is a totally imaginary quadratic extension over a totally real algebraic number field F. The main result of this paper is the following:

THEOREM. *Let F be a totally real algebraic number field of finite degree. Let l be an odd prime which does not divide $w_F\zeta_F(-1)$. Then there exist infinitely many quadratic extensions K/F with the following properties:*

(i) *K is totally imaginary,*

(ii) *the relative class number $h_{K/F}$ of K/F is not divisible by l,*

(iii) *each prime ideal of F over l does not split in K.*

If $F=\boldsymbol{Q}$, this is the result due to Hartung [2], since $w_{\boldsymbol{Q}}\zeta_{\boldsymbol{Q}}(-1)=-2$. In order to get Theorem, we use trace formulas and l-adic representations related to automorphic forms obtained from division quaternion algebras over F.

Let K/F be a totally imaginary quadratic extension. We denote by μ_K^-

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