J. Math. Soc. Japan
Vol. 43, No. 1, 1991

An ergodic control problem arising from the principal eigenfunction of an elliptic operator

By Alain BENSOUSSAN and Hideo NAGAI(*)

(Received Jan. 9, 1989)
(Revised Nov. 27, 1989)

0. Introduction.

Let us consider the following second order quasi-linear partial differential equation:

$$-\frac{1}{2}\Delta v_\alpha + H(x, \nabla v_\alpha) + \alpha v_\alpha = 0 \tag{0.1}$$

with a quadratic growth nonlinear term $H(x, \nabla v_\alpha)$ on ∇v_α, where α is a positive constant. Such kinds of equations on bounded regions with periodic or Neumann boundary conditions have been studied by several authors (cf. Bensoussan-Frehse [3], Gimbert [6], Lasry [8], and Lions [9]) in connection with ergodic control problems, where the asymptotic behaviour of the solution v_α of (0.1) as α tends to 0 is investigated. The problems arise from stochastic control problem (cf. Bensoussan [2]). In those works important steps of the resolution of such problems are to deduce the estimates on the L^∞-norms of αv_α and ∇v_α by using the maximum principle and the Bernstein's method. But similar problems on the whole space have been out of consideration because the method does not work. We may say intuitively that main difficulty to treat such problems on the whole space lies in lack of uniform ergodicity of underlying diffusion processes and it seems to be necessary to employ completely different method.

In the present article we specialize the equation (0.1) to the case where

$$H(x, \nabla v_\alpha) = \frac{1}{2}|\nabla v_\alpha|^2 - V(x) \tag{0.2}$$

but treat it on whole Euclidean space $\boldsymbol{R}^n$. We notice the relationship between the equation (0.1) with (0.2) and the eigenvalue problem of a Schrödinger operator $-(1/2)\Delta + V$ in $L^2(\boldsymbol{R}^n)$:

(*) This author was partially supported by Grant-in-Aid for Scientific Research (No. 62302006), Ministry of Education, Science and Culture.