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Delta-unknotting operation and the second coefficient of the Conway polynomial

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§1. Introduction.

In this paper, we study oriented tame links in the oriented 3-sphere S^3. A Δ-unknotting operation is a local move on an oriented link diagram as indicated in Figure 1.1.

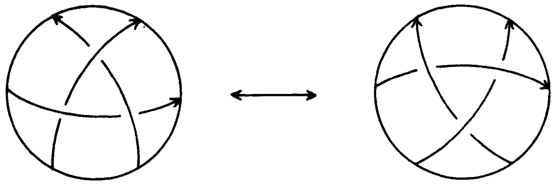

Figure 1.1. Δ-unknotting operation.

In [8], H. Murakami and Y. Nakanishi introduced this notion and proved that every knot can be transformed into a trivial knot by a finite number of Δ-unknotting operations. Let K and K' be oriented knots in S^3. The Δ-Gordian distance from K to K', denoted by $d_G^\Delta(K, K')$, is the minimum number of Δ-unknotting operations which are necessary to deform a diagram of K into that of K'. The Δ-unknotting number of K, denoted by $u^\Delta(K)$, is the Δ-Gordian distance from K to a trivial knot. Then they showed the congruences $d_G^\Delta(K, K') \equiv \mathrm{Arf}(K)-\mathrm{Arf}(K') \pmod 2$ and $u^\Delta(K) \equiv \mathrm{Arf}(K) \pmod 2$ in [8], where $\mathrm{Arf}(K)$ is the Arf invariant of a knot K. Let $a_i(L)$ be the i-th coefficient of the Conway polynomial $\nabla_L(z)$ of a link L. It is known that $a_i(L)$ has a relation to the Casson's invariant ([1], [3]). For the definition and fundamental properties of the Conway polynomial, we refer to [4]. In this paper, we show the following:

THEOREM 1.1. *Let K and K' be two knots with $d_G^\Delta(K, K')=1$. Then, we have*

$$|a_2(K)-a_2(K')| = 1 .$$

As an immediate consequence of Theorem 1.1, we have the following:

COROLLARY 1.2. *For any two knots K and K', the difference $d_G^\Delta(K, K')-|a_2(K)-a_2(K')|$ is a non-negative even integer. In particular the difference $u^\Delta(K)$*