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Mode-conversion of the scattering kernel for the elastic wave equation

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§ 0. Introduction.

Let Ω be an exterior domain in $\boldsymbol{R}^3$ with a smooth and compact boundary $\partial\Omega$. We consider the isotropic elastic wave equation

$$(0.1)\qquad \begin{cases} Lu \equiv (A(\partial_x)-\partial_t^2)u = 0 & \text{in } \boldsymbol{R}\times\Omega, \\ u(t, x) = 0 & \text{on } \boldsymbol{R}\times\partial\Omega, \\ u(0, x) = f_0, \quad \partial_t u(0, x) = f_1 & \text{on } \Omega. \end{cases}$$

Here $u={}^t(u_1, u_2, u_3)$ is the displacement vector and $A(\partial_x)$ is of the form

$$A(\partial_x)u = \mu\Delta u+(\lambda+\mu)\operatorname{grad}(\operatorname{div} u).$$

We assume that the Lamé constants λ and μ satisfy

$$\lambda+\frac{2}{3}\mu > 0 \qquad \text{and} \qquad \mu > 0 .$$

Then, as is shown in Yamamoto [**13**] and Shibata and Soga [**8**], we can develop the scattering theory for (0.1) in a similar way to that in Lax and Phillips [**6**]. Let $k_-(s, \omega)$ and $k_+(s, \omega)\in L^2(\boldsymbol{R}\times S^2)$ $(=\{L^2(\boldsymbol{R}\times S^2)\}^3)$ be the incoming and outgoing translation representations of the initial data $f=(f_0, f_1)$ respectively. The mapping $S: k_-\to k_+$ is called the scattering operator, which is a unitary operator from $L^2(\boldsymbol{R}\times S^2)$ to itself. The scattering operator is represented with a distribution kernel $S(s, \theta, \omega)$ called the scattering kernel:

$$(Sk_-)(s, \theta) = \iint_{\boldsymbol{R}\times S^2} S(s-t, \theta, \omega)k_-(t, \omega)dtd\omega .$$

Note that the scattering kernel $S(s, \theta, \omega)$ is a 3×3-matrix whose components are smooth functions in θ and ω with the value of the distribution in s. The purpose of the present paper is to study singularities of the scattering kernel $S(s, \theta, \omega)$.

The characteristic matrix $A(\xi)$ of the operator $A(\partial_x)$ has the eigenvalues $C_L{}^2|\xi|^2$ and $C_T{}^2|\xi|^2$, where

$$C_L = (\lambda+2\mu)^{1/2}, \qquad C_T = \mu^{1/2} .$$