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The short time asymptotics of the traces of the heat kernels for the magnetic Schrödinger operators

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§ 1. Introduction.

The short time asymptotics of the traces of the heat kernels for the Schrödinger operators without magnetic fields on Euclidean spaces has been considered from the old times mainly because it is directly related to the asymptotic distributions of the eigenvalues by virtue of the Tauberian theorem. See, for example, [**6**], [**8**] and the references therein. Moreover Tamura [**10**] has studied this short time asymptics for the Schrödinger operators only with magnetic fields whose magnitudes grow unboundedly at infinity.

In this paper we will consider the Schrödinger operators H_0 and H on $\boldsymbol{R}^d$, $d\geqq 2$, which are given by

$$H_0=-\frac{1}{2}\Delta+V(x)$$

and

$$H=\frac{1}{2}(-\sqrt{-1}\nabla+A(x))^2+V(x),$$

both of which act on $L^2(\boldsymbol{R}^d)$, and we will study the difference between the short time asymptotics of the traces of the heat kernels for $-H_0$ and $-H$.

We will assume that the scalar potential V is bounded from below and is bounded from below by some polynomial of $|x|$ at infinity. This implies that e^{-tH_0} and e^{-tH} are of the trace class. Then we will see that, if the derivatives $\partial^\alpha A(x)$, $|\alpha|=1, 2$, of the vector potential A grow more slowly than V, the leading term of the asymptotics of the trace of e^{-tH} as t tends to 0 coincides with that of e^{-tH_0}. Therefore such vector potentials or magnetic fields, which are given by $\mathrm{curl}(A(x))$, do not affect the asymptotic behavior of the trace so seriously. Moreover this result says that the leading terms of the asymptotic distributions of the eigenvalues of H_0 and H are identical.

Similar problem has been considered by Odencrantz [**5**] in the case of a uniform magnetic field. This important case will be discussed in detail also in this paper. The main methods used in [**5**] are the canonical order calculus