J. Math. Soc. Japan
Vol. 42, No. 4, 1990

Manifolds which have two projective space bundle structures from the homotopical point of view

Dedicated to Professor Shôrô Araki on his 60th birthday

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(Received July 24, 1989)
(Revised Oct. 9, 1989)

Introduction

E. Sato [2] considered the structure of varieties which have two bundle structures whose fibers are projective spaces. It is interesting to consider this problem from a different point of view. In this paper we consider it from the homotopical point of view.

Let X be a manifold which has the following two bundle structures:

$$CP^r \xrightarrow{i_1} X \xrightarrow{p_1} CP^m, \qquad CP^s \xrightarrow{i_2} X \xrightarrow{p_2} Y,$$

where $r, s, m \geqq 1$ and Y is a manifold. The purpose of this paper is to classify the cohomology ring of X and describe the cohomology ring of Y in terms of that of X. But when we consider this problem from the homotopical point of view, it is difficult to distinguish fiber bundles from fibrations. Hence what we do in this paper is to consider manifolds with two maps $p_1: X \to CP^m$ and $p_2: X \to Y$, where Y is a manifold, whose homotopy fibers are complex projective spaces.

Before we state the results of this paper we list the non-trivial examples to see that there are many examples.

EXAMPLE. (1) By $H_{m,m}$ we denote the Milnor manifold, that is,

$$H_{m,m} = \{([x_0 : \cdots : x_m], [y_0 : \cdots : y_m]) \in CP^m \times CP^m \mid x_0y_0 + \cdots + x_my_m = 0\}.$$

The first and second projections $CP^m \times CP^m \to CP^m$ induce the two projective bundle structures on $H_{m,m}$.

(2)
$$\begin{array}{ccc} CP^n = U(n+1)/U(1)\times U(n) & & U(2)/U(1)\times U(1) = CP^1 \\ \downarrow & & \downarrow \\ U(n+2)/U(1)\times U(1)\times U(n) & = & U(n+2)/U(1)\times U(1)\times U(n) \\ \downarrow & & \downarrow \\ CP^{n+1} = U(n+2)/U(1)\times U(n+1) & & U(n+2)/U(2)\times U(n). \end{array}$$