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The nonexistence of expansive homeomorphisms of Suslinian continua

Dedicated to Professor Ryōsuke Nakagawa on his 60th birthday

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1. Introduction.

All spaces under consideration are assumed to be metric. By a *continuum*, we mean a compact *connected* nondegenerate space. Let X be a compact metric space with metric d. A homeomorphism f of X is called *expansive* if there exists $c>0$ (called an *expansive constant* for f) such that if x and y are different points of X, then there is an integer n such that $d(f^n(x), f^n(y))>c$. Expansiveness does not depend on the choice of metric of X. We are interested in the following problem: What kinds of continua admit expansive homeomorphisms? Here, we consider this problem from a point of view of continuum theory.

Concerning the above problem, the following results are well known.

(i) Each compact metric space which admits an expansive homeomorphism is finite-dimensional ([**12**]).

(ii) The Cantor set, the p-adic solenoids ($p\geqq 2$) and compact orientable surfaces of positive genus admit expansive homeomorphisms ([**13**], [**14**] and [**16**]). There are solenoidal groups which admit no expansive automorphisms (see [**17**, Remark 2, p. 102] and [**18**, Theorem 3, p. 30]).

(iii) The shift homeomorphism of the inverse limit of every continuous surjection of an arc is not an expansive homeomorphism ([**3**] and [**4**]).

(iv) There are no expansive homeomorphisms on the 2-sphere ([**5**]).

(v) If X is a Peano continuum in the plane, or X is a Peano continuum which contains a 1-dimensional AR neighborhood, then X does not admit an expansive homeomorphism ([**1**], [**4**], [**6**], [**7**] and [**11**]).

(vi) There are no expansive homeomorphisms on hereditarily decomposable tree-like (or circle-like) continua ([**8**] and [**9**]).

(vii) There is a continuum in the plane which admits an expansive homeo-

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