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Classification of totally real 3-dimensional submanifolds of $S^6(1)$ with $K \geqq 1/16$

By F. DILLEN[(*)], L. VERSTRAELEN and L. VRANCKEN[(*)]

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1. Introduction.

It is well-known that a 6-dimensional sphere S^6 does not admit any Kaehler structure. However, using the Cayley algebra, a natural almost complex structure J can be defined on S^6 considered as a hypersurface in $\boldsymbol{R}^7$ which itself is viewed as the set of the purely imaginary Cayley numbers. And, together with the standard metric g on S^6, this almost complex structure J determines a *nearly Kaehler* structure in the sense of A. Gray [**G2**]. In Section 2, we recall the construction of this structure working with the 6-dimensional unit sphere $S^6(1)$, (of radius and constant curvature 1).

With respect to the almost complex structure J on $S^6(1)$, two natural particular types of submanifolds M can be investigated: those which are *almost complex* (i.e. for which the tangent space of M at each point is invariant under the action of J) and those which are *totally real* (i.e. for which the tangent space of M at each point is mapped into the normal space at that point by J). The almost complex submanifolds M of the nearly Kaehler $S^6(1)$ are, as the invariant submanifolds of Kaehlerian manifolds, automatically minimal and even dimensional, and therefore of dimension 2 or 4. Moreover, A. Gray [**G1**] showed that there do not exist 4-dimensional almost complex submanifolds in $S^6(1)$. So, for this case, only the almost complex surfaces of $S^6(1)$ need to be studied. Curvature properties for such surfaces were first obtained by K. Sekigawa [**Se**]. As follows at once from their definition, for the other case, only 2- and 3-dimensional totally real submanifolds can occur in $S^6(1)$. N. Ejiri [**E1**] proved that every 3-dimensional totally real submanifold of $S^6(1)$ is orientable and minimal, and he first investigated curvature conditions on such manifolds. The 3-dimensional totally real submanifolds of $S^6(1)$ were also considered, for instance, by H. Bl. Lawson Jr. and R. Harvey [**H-L**] in their study of calibrated geometries, and by K. Mashimo [**M2**] from the viewpoint of homogeneous manifolds.

(*) Research Assistant of the Belgian National Science Foundation.