

A generalized Pohozaev identity and its applications

By Nichiro KAWANO¹⁾, Wei-Ming NI²⁾
and Shoji YOTSUTANI³⁾

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§ 1. Introduction.

In this paper we establish a generalized Pohozaev identity and its variant for the radial solutions of the following quasilinear elliptic equation,

$$(1.1) \quad \operatorname{div}(A(|Du|)Du) + f(|x|, u) = 0$$

in \mathbf{R}^n , where Du is the gradient of u , $f(|x|, u)$ and $A(p)$ are given functions. The Pohozaev identity is useful to investigate the existence and non-existence of the ground state of (1.1). By a ground state we mean a positive solution u in \mathbf{R}^n , which tends to zero at ∞ .

The Pohozaev identity was used by Pohozaev [15] in 1965 to show the non-existence of non-trivial solutions of non-linear eigenvalue problems for semi-linear elliptic equations. Identities of this kind were first discovered by Rellich [17] in 1940 in his study of the first eigenvalue of Δ , and by Nehari [5] in 1960. The idea was applied to investigate the properties of solutions for non-linear elliptic equations (see, e.g., [1], [2], [3], [4], [6], [7], [8], [9], [10], [11], [12], [13], [14], [16]). Especially, Ding and Ni [2] found that the Pohozaev-type identity is useful to get the non-existence theorems for the ground state in the anomalous case, $f_u(|x|, 0) = 0$, by employing suitable change of variables. Recently, Ni and Serrin [9, 10, 11] established some generalized Pohozaev identities and used them to investigate the solutions of the quasilinear elliptic equations,

$$(1.2) \quad \operatorname{div}(A(|Du|)Du) + f(u) = 0.$$

They extend the argument employed by Ding and Ni to the quasilinear case. Their results are sharp, however their arguments are tricky and difficult. Our

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