J. Math. Soc. Japan
Vol. 42, No. 3, 1990

Homogeneous Kähler manifolds of non-degenerate Ricci curvature

Dedicated to Professor N. Tanaka on his 60th birthday

By Kazufumi NAKAJIMA

(Received July 11, 1988)
(Revised July 27, 1989)

Introduction.

Let M be a connected homogeneous Kähler manifold. Denote by $\mathrm{Aut}(M)$ the group of all holomorphic isometries of M. Let G be a connected subgroup of $\mathrm{Aut}(M)$ acting transitively on M and K the isotropy subgroup of G at a point of M. We denote by $\mathfrak{g}$ and $\mathfrak{k}$ the Lie algebras of G and K respectively. Then there correspond to the invariant complex structure and the Kähler form of M a linear endomorphism j of $\mathfrak{g}$ and a skew-symmetric bilinear form ρ on $\mathfrak{g}$ such that $(\mathfrak{g}, \mathfrak{k}, j, \rho)$ becomes an effective Kähler algebra. (For the definition of a Kähler algebra, see § 1.)

According to Vinberg and Gindikin [8], the Kähler algebra $(\mathfrak{g}, \mathfrak{k}, j, \rho)$ is called *non-degenerate* if there exists a linear form ω on $\mathfrak{g}$ such that $\rho=d\omega$ ([8]), where the operator d means the exterior differentiation under the identification of p-forms on $\mathfrak{g}$ with left invariant p-forms on the Lie group G. Note that if the Kähler algebra $(\mathfrak{g}, \mathfrak{k}, j, \rho)$ is non-degenerate, then the system $(\mathfrak{g}, \mathfrak{k}, j)$ becomes a j-algebra. (For the definition of a j-algebra, see § 2.)

The purpose of the present paper is to investigate the structure of j-algebras and prove the following

THEOREM. *Let $M=G/K$ be a connected homogeneous Kähler manifold where G is a subgroup of* $\mathrm{Aut}(M)$. *Then the Ricci curvature of M is non-degenerate if and only if the corresponding Kähler algebra* $(\mathfrak{g}, \mathfrak{k}, j, \rho)$ *is non-degenerate.*

We explain our method. By [3] every connected homogeneous Kähler manifold M is a holomorphic fiber bundle over a homogeneous bounded domain in which the fiber is the product of a flat homogeneous Kähler manifold and a compact simply connected homogeneous Kähler manifold. Recall that the Ricci tensor of M corresponds to the canonical hermitian form introduced by Koszul [4] and it is expressed in terms of the Kähler algebra $(\mathfrak{g}, \mathfrak{k}, j, \rho)$. Then by a simple calculation, we can see in § 1 that *if the Ricci tensor of M is non-*