J. Math. Soc. Japan
Vol. 42, No. 3, 1990

# Periods of cusp forms associated to loxodromic elements of $b$-groups

Dedicated to Michio Kuga* on his sixtieth birthday

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(Received Jan. 30, 1989)
(Revised July 18, 1989)

The purpose of this note is to explore the periods of cusp forms associated to loxodromic elements of $b$-groups (function groups with simply connected invariant components). Let $\alpha$ and $\beta$ be two distinct points in $\boldsymbol{C}\cup\{\infty\}$. Let

$$g_{\alpha,\beta}(z)=\frac{\alpha-\beta}{(z-\alpha)(z-\beta)}, \qquad z\in\boldsymbol{C}\cup\{\infty\}. \tag{0.1}$$

Let $\Gamma$ be a finitely generated non-elementary Kleinian group with region of discontinuity $\Omega=\Omega(\Gamma)$ and limit set $\Lambda=\Lambda(\Gamma)$. Fix an integer $q\geqq 2$ and let $\boldsymbol{A}_q(\Omega, \Gamma)$ denote the space of cusp forms for $\Gamma$ of weight $(-2q)$ (or cusp $q$-forms, for short). For $A\in\Gamma$, a loxodromic (including hyperbolic) element with attractive fixed point $\alpha$ and repulsive fixed point $\beta$, we introduce the relative Poincaré series

$$\varphi_A(z)=\sum_{\gamma\in\Gamma_0\backslash\Gamma} g_{\alpha,\beta}^q(\gamma(z))\gamma'(z)^q, \qquad z\in\Omega, \tag{0.2}$$

where $\Gamma_0=\langle A\rangle$, the cyclic group generated by $A$. It was shown in [**K3**] that $\varphi_A\in\boldsymbol{A}_q(\Omega, \Gamma)$.

Assume now that $\Gamma$ is a $b$-group and $\Delta$ is a simply connected invariant component of $\Gamma$ (that is, of $\Omega(\Gamma)$). If $B$ is a loxodromic element of $\Gamma$ with attractive fixed point $a$ and repulsive fixed point $b$, then the *period* $L_B(\varphi)$ *of* $\varphi\in\boldsymbol{A}_q(\Omega, \Gamma)$ *along* $B$ is defined by

$$L_B(\varphi)=\int_{z_0}^{Bz_0} g_{a,b}^{1-q}(z)\varphi(z)dz. \tag{0.3}$$

The integral is independent of the point $z_0$ in $\Delta$ as long as the path of integration is restricted to lie in $\Delta$. The period of $\varphi$ depends, of course, only on $\varphi|\Delta$ (the space of restrictions of cusp forms to $\Delta$ will be denoted by $\boldsymbol{A}_q(\Delta, \Gamma)$).

The periods are conjugation invariant in the following sense. Let $C_1$ and $C_2$ be two arbitrary elements of $PSL(2, \boldsymbol{C})$ with the property $C_1\Gamma C_1^{-1}=C_2\Gamma C_2^{-1}$,

Research partially supported by NSF grant DMS 8701774.
* Professor Kuga died on February 14, 1990.