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The quasi KO-homology types of the stunted real projective spaces

Dedicated to Professor Akio Hattori on his sixtieth birthday

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0. Introduction.

Let E be an associative ring spectrum with unit, and X, Y be CW-spectra. We say that X is *quasi E_*-equivalent to Y* if there exists a map $h: Y\to E\wedge X$ such that the composite $(\mu\wedge 1)(1\wedge h): E\wedge Y\to E\wedge X$ is an equivalence where $\mu: E\wedge E\to E$ stands for the multiplication of E. In this case we write $X\underset{E}{\sim}Y$, and we call such a map $h: Y\to E\wedge X$ a quasi E_*-equivalence. We shall be concerned with the quasi KO_*-equivalence where KO is the real K-spectrum. In [**Y2**] we have determined the quasi KO_*-types of the real projective n-spaces RP^n. The purpose of this note is to determine the quasi KO_*-types of the stunted real projective spaces RP^n/RP^m as a continuation of [**Y2**].

In order to describe our main result precisely we have to introduce some elementary suspension spectra with three or four cells (see [**Y3, Y4**]). The Moore spectrum SZ/n of type Z/n is constructed by the cofiber sequence $\Sigma^0\xrightarrow{n}\Sigma^0\xrightarrow{i}SZ/n\xrightarrow{j}\Sigma^1$. Let M_{2m} and V_{2m} denote the cofibers of the maps $i\eta:\Sigma^1\to SZ/2m$ and $i\bar{\eta}:\Sigma^1SZ/2\to SZ/m$ respectively. Here $\eta:\Sigma^1\to\Sigma^0$ stands for the stable Hopf map of order 2 and $\bar{\eta}:\Sigma^1SZ/2\to\Sigma^0$ its extension satisfying $\bar{\eta}i=\eta$. The complex K-spectrum KU possesses the conjugation $t: KU\to KU$ which gives rise to an involution t_* on KU_*X for any CW-spectrum X. By comparing KU_*RP^n with KU_*M_{2m} or KU_*V_{2m} as an abelian group with involution, and then by characterizing a CW-spectrum X which admits the same quasi KO_*-type as M_{2m} or V_{2m}, we have established the following determination [**Y2**, Theorem 5] (cf. [**F**]).

THEOREM 1. *Σ^1RP^n is quasi KO_*-equivalent to $SZ/2^{4r}$, $M_{2^{4r}}$, $V_{2^{4r+1}}$, $\Sigma^4\vee V_{2^{4r+1}}$, $V_{2^{4r+2}}$, $M_{2^{4r+2}}$, $SZ/2^{4r+3}$, $\Sigma^0\vee SZ/2^{4r+3}$ according as $n=8r$, $8r+1$, $\cdots$, $8r+7$.*

Let M'_{2m} and MP_{2m} denote the cofibers of the maps $\eta j: SZ/2m\to\Sigma^0$ and $i\eta\vee\tilde{\eta}:\Sigma^1\vee\Sigma^2\to SZ/2m$ respectively. Here $\tilde{\eta}:\Sigma^2\to SZ/2m$ stands for a coexten-