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Reduction for Painlevé equations at the fixed singular points of the second kind

Dedicated to Professor Tosihusa Kimura on his 60th birthday

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§ 1. Introduction.

This paper gives a simple reduction theorem for Painlevé equations near the fixed singular points of the second kind in the framework of Hamiltonian mechanics.

It is known that each Painlevé equation P_J (J=I, ⋯, VI) is equivalent to a Hamiltonian system: $d\lambda/dt=\partial H_J/\partial\mu$, $d\mu/dt=-\partial H_J/\partial\lambda$, where the Hamiltonian function $H_J=H_J(t, \lambda, \mu)$ is a polynomial of λ and μ of which the coefficients are rational functions of t ([**14**]). We call these Hamiltonian systems Painlevé systems. Then the fixed singular points are formally classified as follows: a fixed singular point of a Painlevé equation is *of the first kind* or *of the second kind* if Poincaré rank of the corresponding Painlevé system at the point is zero or positive respectively.

We want to construct a 2-parameter family of solutions of each Painlevé system at each fixed singular point, in other words, to obtain a local biholomorphic transformation which reduces it to a solvable system. As is well known, concerning the construction of an n-parameter family of solutions of an n-system at a fixed singular point, we have a general theorem by J. Malmquist under the so-called Poincaré's condition ([**12**], [8]). However, we can not apply the theorem to Painlevé systems because Poincaré's condition is completely violated for them.

Recently, having been stimulated by the idea of M. Iwano ([**9**]), several authors have obtained 2-parameter families of solutions of Painlevé systems at the fixed singular points of the second kind ([**16**], [**15**], [**19**], [**20**]). Their works especially those by S. Yoshida explain, from a general point of view, the reason why the formal transformations for Painlevé systems without

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