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Necessary and sufficient conditions for optimality in nonlinear distributed parameter systems with variable initial state

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1. Introduction.

In this paper, we consider a nonlinear distributed control system, with time varying control constraints and an initial condition which is not determined by an a priori given function, but instead it is assumed to belong to a certain specified set (Lions [**5**] calls them "systems with insufficient data"). The cost criterion is a general convex integral functional.

Using the Dubovitski-Milyutin formalism, we are able to obtain a necessary and sufficient condition for the existence of an optimal solution. A very comprehensive presentation of the Dubovitski-Milyutin theory can be found in the monograph of Girsanov [**3**]. Our result extends Theorem 2.1 of Lions [**5**], since we allow for nonlinear dynamics and a nonquadratic cost criterion.

2. Preliminaries.

The mathematical setting is the following. Let $T=[0, b]\subseteq \boldsymbol{R}_+$ (a bounded time interval) and H a separable Hilbert space. Also let $X\subseteq H$ be a subspace of H carrying the structure of a separable reflexive Banach space, which imbeds continuously and densely into H. Identifying H with its dual (pivot space), we have $X\hookrightarrow H\hookrightarrow X^*$, with all embeddings being continuous and dense. Such a triple (X, H, X^*) of spaces is sometimes called "Gelfand triple" or "spaces in normal position". By $\|\cdot\|$ (resp. $|\cdot|$, $\|\cdot\|_*$) we will denote the norm of X (resp. of H, X^*). Also by $(\cdot, \cdot)$ we will denote the inner product in H and by $\langle\cdot, \cdot\rangle$ the duality brackets for the pair (X, X^*). The two are compatible in the sense that if $x\in X\subseteq H$ and $h\in H\subseteq X^*$, we have $(x, h)=\langle x, h\rangle$. Also let Y be another separable Banach space modelling the control space. By $P_{fc}(Y)$ we will denote the nonempty, closed, convex subsets of Y.

The optimal control problem under consideration is the following:

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