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Backward Itô's formula for sections of a fibered manifold

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1. Introduction.

When a stochastic differential equation on a C^{∞} manifold generates a stochastic flow of diffeomorphisms of the manifold, some geometric backward Itô's formulas related to the stochastic flow are known for tensor fields on the manifold [7], [**10**]. On the other hand, a forward (usual) Itô's formula was obtained for (local) sections of a fiber bundle [**1**]. It is, therefore, desirable to get a backward stochastic formula generalized for (local) sections of a fiber bundle or, more generally, of a fibered manifold.

The main purpose of the present paper is to obtain a backward stochastic formula, which we will also call backward Itô's formula, for (local) sections of a fibered manifold (Theorem 3.2) with the use of backward stochastic calculus ([**10**]). As a corollary, we obtain a backward Itô's formula for sections of a vector bundle (Corollary 4.1). Then, using this formula, we treat certain backward and forward differential equations for sections of a vector bundle (Corollaries 4.3 and 4.4).

Although the formula in Theorem 3.2 is applicable to C^{∞} sections of a general C^{∞} fibered manifold, we are chiefly concerned with sections of fiber bundles; we give applications to the study of the behavior (with respect to the initial-time parameter) of a time-dependent random C^{∞} distribution of a C^{∞} manifold and to backward and forward differential equations for second order (possibly degenerate) linear differential operators on C^{∞} functions on a C^{∞} manifold (§5). These applications are done by noting that a C^{∞} distribution of a C^{∞} manifold can be regarded as a section of a Grassmann bundle ([**3**]), and that each second order linear differential operator on C^{∞} functions on a C^{∞} manifold can be identified with a section of a certain vector bundle associated with the bundle of second order frames of the manifold.

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