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Quasi-periodicity of bounded solutions to some periodic evolution equations

Dedicated to Professor Hiroshi Fujita on the occasion of his sixtieth birthday

By Alain HARAUX and Mitsuharu ÔTANI

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Introduction.

Let H be a real Hilbert space with norm denoted by $\|\cdot\|$ and inner product by $\langle\ ,\ \rangle$. For any $t\in\boldsymbol{R}$, let $A(t): D[A(t)]\to H$ be a maximal monotone operator. We consider the evolution equation

$$u'(t)+A(t)u(t)\ni 0. \tag{0.1}$$

In the sequel, we denote by $(C_t)_{t\in\boldsymbol{R}}$ the closure in H of the domain $D[A(t)]$. It is well known that C_t is *convex* (cf. e.g. [**5**]).

Under several different types of technical assumptions, it is possible to define for any $s\in\boldsymbol{R}$ and any $x\in C_s$ a unique "weak" solution $u(t)$ of (0.1) on $[s, +\infty[$ such that $u(s)=x$. In general, u is not differentiable and is constructed by some approximation procedure (cf. e.g. [**1, 2, 4, 5, 6, 14, 17, 20**]).

In all the cases in which this construction is possible, u is given by the formula

$$\forall t\geqq s, \qquad u(t)=E(s, t)x \tag{0.2}$$

where $E(s, t): C_s\to H$ is defined for $t\geqq s$ and satisfies the following properties

$$\forall s\in\boldsymbol{R},\ \forall x\in C_s,\ \forall t\geqq s, \qquad E(s, t)x\in C_t. \tag{0.3}$$

$$\forall s\in\boldsymbol{R},\ \forall x\in C_s,\ \forall t_2\geqq t_1\geqq s, \qquad E(s, t_2)x=E(t_1, t_2)E(s, t_1)x. \tag{0.4}$$

$$\forall s\in\boldsymbol{R},\ \forall t\geqq s,\ \forall x\in C_s,\ \forall y\in C_s, \qquad \|E(s, t)x-E(s, t)y\|\leqq\|x-y\|. \tag{0.5}$$

Let J be a closed interval of $\boldsymbol{R}$. We say that a function $u\in C(J, H)$ is *a solution of* (0.1) *on* J if u satisfies

$$\forall s\in\boldsymbol{R},\ \forall t\in J,\ t\geqq s, \qquad u(t)=E(s, t)u(s). \tag{0.6}$$

We say that u is a *strong solution of* (0.1) *on* J if $u\in W^{1,1}(K, H)$ for any compact interval $K\subset J$ and for almost all $t\in K$, $u(t)\in D[A(t)]$ and $u'(t)\in -A(t)u(t)$.

In this paper, we are mainly interested in the case where $A(t)$ is periodic,