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Notes on C^0 sufficiency of quasijets

Dedicated to Professor Masahisa Adachi on his 60th birthday

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Let $\mathcal{E}_{[k]}(n, 1)$ be the set of C^k function germs: $(\boldsymbol{R}^n, 0)\to(\boldsymbol{R}, 0)$ for $k=1, 2, \cdots, \infty, \omega$, and let $\mathcal{H}(n, 1)$ be the set of holomorphic function germs: $(\boldsymbol{C}^n, 0)\to(\boldsymbol{C}, 0)$. If for two function germs $f, g\in\mathcal{E}_{[k]}(n, 1)$ (resp. $\mathcal{H}(n, 1)$) there exists a local homeomorphism $\sigma:(\boldsymbol{R}^n, 0)\to(\boldsymbol{R}^n, 0)$ (resp. $\sigma:(\boldsymbol{C}^n, 0)\to(\boldsymbol{C}^n, 0)$) such that $f=g\circ\sigma$, we say that f is *C^0-equivalent* to g and write $f\sim g$. We shall not distinguish between germs and their representatives.

Consider the polynomial function $f:(\boldsymbol{R}^2, 0)\to(\boldsymbol{R}, 0)$ defined by

$$f(x, y)=x^3+3xy^{20}+y^{29}.$$

Then we see that

$$x^3+3xy^{20}\overset{\text{(i)}}{\sim}x^3+3xy^{20}+y^{29}\overset{\text{(ii)}}{\sim}x^3+y^{29}.$$

Here we interpret the above equivalences as follows (see [**6**], Example 4.3 also):

(i) Put $w=j^{21}f(0)=x^3+3xy^{20}$. Then w is C^0-equivalent to f. This follows from the Kuiper-Kuo theorem (see Lemma 5 in §3).

(ii) Put $z=x^3+y^{29}$. Then z is C^0-equivalent to f. Since z is weighted homogeneous of type (1/3, 1/29) with a finite codimension and the weight of the term $3xy^{20}$ is $1/3+20/29>1$ (see V.I. Arnol'd [**1**]).

In the complex case, the equivalence (i) does not hold. For w is weighted homogeneous of type (1/3, 1/30) with an isolated singularity and the weight of the term y^{29} is $29/30<1$. Furthermore $y^{29}\notin\mathfrak{M}(\partial w/\partial x, \partial w/\partial y)$. Therefore w is not C^0-equivalent to $w+y^{29}=f$ (see M. Suzuki [**16**] or A.N. Varčenko [**18**]). (Of course, we can also see this directly by considering the C^0-type of $w^{-1}(0)$ and $f^{-1}(0)$, as germs at $0\in\boldsymbol{C}^2$.) Even in the real case, the equivalence (i) does not hold, if we replace plus by minus (i.e. $w=x^3-3xy^{20}$).

PROBLEM. *Is there a unified discription for explaining the above interpretations?*

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