J. Math. Soc. Japan
Vol. 42, No. 2, 1990

Consistency of Menas' conjecture

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(Received March 7, 1989)

In this paper we will prove the consistency of the following conjecture of Menas [8] with ZFC. Menas' conjecture: *For every regular uncountable cardinal κ and λ a cardinal $>\kappa$, if X is a stationary subset of $\mathcal{P}_\kappa\lambda$ then X splits into $\lambda^{<\kappa}$ many disjoint stationary subsets.* We will prove the consistency of the conjecture by showing that it holds in L, the class of constructible sets.

Baumgartner and Taylor [1] have shown the consistency of the failure of Menas' conjecture with ZFC. Thus we can conclude that Menas' conjecture is independent of ZFC. Throughout this paper we let κ denote a regular uncountable cardinal and λ a cardinal $>\kappa$.

Baumgartner and DiPrisco proved that if $0^\#$ does not exist then every stationary subset of $\mathcal{P}_\kappa\lambda$ splits into λ many disjoint stationary subsets. In [6], we have proved the following, strengthening their result slightly using generic ultrapowers.

THEOREM 1. *If there is a stationary subset of $\mathcal{P}_\kappa\lambda$ which does not split into λ many disjoint stationary subsets, then $b^\#$ exists for every bounded subset b of λ.*

The proof of Theorem 1 was based on the following two results.

THEOREM 2 (Foreman [2]). *If I is a countably complete λ^+-saturated ideal on $\mathcal{P}_\kappa\lambda$ then I is precipitous.*

THEOREM 3 ([6]). *If there is a precipitous ideal on $\mathcal{P}_\kappa\lambda$ then $b^\#$ exists for every bounded subset b of λ.*

Let $\mathrm{NS}(\kappa, \lambda)$ denote the nonstationary ideal on $\mathcal{P}_\kappa\lambda$. Thus $\mathrm{NS}(\kappa, \lambda)$ is a κ-complete normal idea. If X is a stationary subset of $\mathcal{P}_\kappa\lambda$ which does not split into λ many disjoint stationary subsets then $\mathrm{NS}(\kappa, \lambda)|X$ is a λ-saturated κ-complete normal ideal on $\mathcal{P}_\kappa\lambda$ where

$$\mathrm{NS}(\kappa, \lambda)|X = \{Y \subseteq \mathcal{P}_\kappa\lambda : Y \cap X \in \mathrm{NS}(\kappa, \lambda)\}.$$

Thus by Theorem 2, the existence of a stationary subset of $\mathcal{P}_\kappa\lambda$ which does not split into λ many disjoint stationary subsets implies the existence of a precipitous ideal on $\mathcal{P}_\kappa\lambda$.