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Extension of minimal immersions of spheres into spheres

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§1. Introduction.

The purpose of the present study is to get isometric minimal immersions of $S^{m+k}(1)$ into spheres which are extensions of isometric minimal immersions of $S^{m}(1)$ into spheres and to find some properties of such immersions.

Let $S^{n-1}(r)$ denote the sphere of radius r centered at the origin in $\boldsymbol{R}^{n}$. An isometric minimal immersion $f_{m,s}: S^{m}(1)\rightarrow S^{n-1}(r)$ is expressed by

$$f_{m,s}(u)=\sum_{A=1}^{n} f^{A}(u)\tilde{e}_{A}$$

where $\{\tilde{e}_{1}, \cdots, \tilde{e}_{n}\}$ is an orthonormal basis of $\boldsymbol{R}^{n}$ and $u\in S^{m}(1)$. By a theorem of Takahashi [7] f^{A} $(A=1, \cdots, n)$ are spherical harmonics of degree s,

$$\Delta f^{A}=\lambda_{s}f^{A}, \qquad \lambda_{s}=s(s+m-1).$$

Let $\{e_{1}, \cdots, e_{m+1}\}$ be an orthonormal basis of $\boldsymbol{R}^{m+1}$ and $S^{m}(1)$ be the unit sphere in $\boldsymbol{R}^{m+1}$ so that we can put $u=u^{i}e_{i}$ using summation convention. To an eigenfunction f of Δ with $\Delta f=\lambda_{s}f$, there corresponds a unique harmonic polynomial

$$F=F_{i_{1}\cdots i_{s}}x^{i_{1}}\cdots x^{i_{s}}$$

of degree s such that

$$f(u)=F_{i_{1}\cdots i_{s}}u^{i_{1}}\cdots u^{i_{s}}.$$

The harmonic polynomial F then is viewed as a symmetric harmonic tensor of degree s, satisfying

i) $F(v_{1}, \cdots, v_{s})$ is symmetric in $v_{1}, \cdots, v_{s}$

ii) $\sum_{i}F(e_{i}, e_{i}, v_{3}, \cdots, v_{s})=0$

where $v_{1}, \cdots, v_{s}\in\boldsymbol{R}^{m+1}$.

Thus, to an isometric minimal immersion $f_{m,s}$ there corresponds a set of n symmetric harmonic tensors $\{F^{1}, \cdots, F^{n}\}$. Let $V(m, s)$ denote the vector space of symmetric harmonic tensors of degree s on $\boldsymbol{R}^{m+1}$. Then we know that $\dim V(m, s)=n(m, s)$ is given by

$$n(m, s)=(2s+m-1)(s+m-2)!/(s!(m-1)!),$$