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Joint spectra of strongly hyponormal operators on Banach spaces

Dedicated to Professor Emeritus Eiitiro Homma with respect

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1. Introduction.

The joint spectrum for a commuting n-tuple in functional analysis has its origin in functional calculus which appeared in J. L. Taylor's paper [**23**] in 1970. In the case of operators on Hilbert spaces, in [**25**] F.-H. Vasilescu characterized the joint spectrum for a commuting pair and in [**11**] R. Curto did it for a commuting n-tuple.

For those on a Banach space, in [**18**] and [**19**] A. McIntosh, A. Pryde and W. Ricker characterized the joint spectrum for a strongly commuting n-tuple of operators. In [**5**] M. Chō proved that the joint spectrum for such an n-tuple is the joint approximate point spectrum of it.

The aim of this paper is to give a characterization of the joint spectrum for a doubly commuting n-tuple of strongly hyponormal operators on a uniformly convex and uniformly smooth space.

Let E^n be the complex exterior algebra on n-generators $e_1, \cdots, e_n$ with product $\wedge$. Then E^n is graded: $E^n=\oplus_{k=-\infty}^{\infty}E_k^n$, where $E_k^n \wedge E_1^n \subset E_{k+1}^n$ and $\{e_{j_1}\wedge\cdots\wedge e_{j_k} : 1\leqq j_1<\cdots<j_k\leqq n\}$ is a basis for $E_k^n (k\geqq 1)$, while $E_0^n \cong \boldsymbol{C}$ and $E_k^n=(0)$ for $k<0$ and $k>n$. Let X be a complex Banach space and $\boldsymbol{T}=(T_1, \cdots, T_n)$ be a commuting n-tuple of bounded linear operators on X. Let $E_k^n(X)=E_k^n\otimes X$ and define $D_k^{(n)}: E_k^n(X)\to E_{k-1}^n(X)$ by $D_k^{(n)}(x\otimes e_{j_1}\wedge\cdots\wedge e_{j_k})=\sum_{i=1}^k(-1)^{i+1}T_{j_i}x\otimes e_{j_1}\wedge\cdots\wedge \check{e}_{j_i}\wedge\cdots\wedge e_{j_k}$ when $k>0$ (here $\check{}$ means deletion), and $D_k^{(n)}=0$ when $k\leqq 0$ and $k>n$. A straightforward computation shows that $D_k^{(n)}\circ D_{k+1}^{(n)}=0$ for all k, so that $\{E_k^n(X), D_k^{(n)}\}_{k\in \boldsymbol{Z}}$ is a chain complex, called the Koszul complex for $\boldsymbol{T}=(T_1, \cdots, T_n)$ and denoted by $E(X, \boldsymbol{T})$. Of course, the mapping $D_k^{(n)}$ depends on $\boldsymbol{T}=(T_1, \cdots, T_n)$. We denote it by $D_k^{(n)}(\boldsymbol{T})$, if necessary.

We define $\boldsymbol{T}=(T_1, \cdots, T_n)$ to be invertible in case its associated Koszul complex is exact (that is, $\mathrm{Ker}(D_k^{(n)})=R(D_{k+1}^{(n)})$ for all k). The Taylor spectrum $\sigma(\boldsymbol{T})$ for $\boldsymbol{T}=(T_1, \cdots, T_n)$ is the set of $z\in \boldsymbol{C}^n$ such that $\boldsymbol{T}-z=(T_1-z_1, \cdots, T_n-z_n)$ is not invertible.