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A remark on the existence of a diffusion process with non-local boundary conditions

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Introduction.

It is known that a diffusion process on a domain D with smooth boundary is determined by a pair of analytical data (A, L), where A is a second order differential operator of elliptic type (possibly degenerate) and L is a Wentzell's boundary condition which consists of the sum of a second order differential operator and non-local terms. (For the precise definition see § 1.) The problem of constructing the diffusion from a pair (A, L) has been discussed by many authors. Analytically, K. Sato and T. Ueno [**12**] laid a fundamental route and following it, Bony-Courrège-Priouret [**2**] and Taira [**14**] succeeded in very general cases. In their manner, one constructs a Feller semigroup (and hence the transition function) on $\bar{D}$ via Hille-Yosida semigroup theory to dispose the diffusion.

On the other hand, the construction of a semigroup can be carried out directly by probabilistic methods, which have an advantage to permit the degeneracy of A. That is, by using stochastic calculus or the martingale method. See Ikeda [**9**], Watanabe [**17**], Stroock-Varadhan [**13**], Anderson [**1**] and Cattiaux [**5**, **6**]. Apart from this, a direct construction of path functions (and hence the diffusion process) by using the notion of Poisson point process of Brownian excursion was succeeded by Watanabe [**18**]. See also Ikeda-Watanabe [**10**] and Takanobu-Watanabe [**16**].

Although we can construct diffusions as the functionals on Wiener-Poisson space as above, we have another task left to verify regularity results, for example, statements about transition functions. Returning to the viewpoint of analysis, one way to treat this problem in the case with non-local boundary conditions will be the use of the theory of pseudodifferential operators developed by Hörmander [**8**], Boutet de Monvel [**3**] et al. It is natural to make such a study since the class of pseudodifferential operators includes a wide class of significant non-local operators (cf. Cancelier [**4**]).

Here we shall afford a concrete example in this framework. That is, the pair (A, L) is given by a second order differential operator of uniformly elliptic