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Generalized Faber expansions of hyperfunctions on analytic curves

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1. Introduction.

Generalized functions and hyperfunctions have been introduced and studied by many people using different approaches. In the work of Köthe [**11**], [**12**], Grothendieck [**6**], Gelfand and Silov [**3**], [**4**], Schwartz [**19**], Roumieu [**15**] and Lions and Magenes [**13**], they were viewed as continuous linear functionals acting on some test-function spaces which are usually the inductive limits of sequences of normed spaces. However, in the work of Sato [**17**], hyperfunctions were viewed more as algebraic objects pertaining to the boundary values of holomorphic functions than as continuous linear functionals. In the case of the real line $\boldsymbol{R}$, the notion of a hyperfunction in Sato's theory is very simple; a hyperfunction on $\boldsymbol{R}$ is defined by a holomorphic function on $\boldsymbol{C}-\boldsymbol{R}$ where $\boldsymbol{C}$ is the complex plane. And two such functions represent the same hyperfunction if and only if their difference is holomorphic on $\boldsymbol{C}$, hence on $\boldsymbol{R}$. More generally, if I is an open subset of $\boldsymbol{R}$ and V is an open subset of $\boldsymbol{C}$ containing I and in which I is relatively closed, then the module of hyperfunctions on I is defined as the quotient module $\mathcal{H}(V-I)/\mathcal{H}(V)$ where $\mathcal{H}(V-I)$ and $\mathcal{H}(V)$ are the complex modules of locally holomorphic functions on $V-I$ and V respectively.

Sato's hyperfunctions have been defined on more general sets in the complex plane such as curves and have also been generalized to higher dimensions using sheaf theory.

On the unit circle ∂D, hyperfunctions were first characterized by Köthe [**11**], [**12**] as continuous linear functionals acting on the linear space of holomorphic complex-valued functions on ∂D when provided with a certain locally convex topology. Using different approaches, Sato [**16**] and Johnson [8] were able to find a very interesting characterization of hyperfunctions on the unit circle in terms of Fourier series. They showed that $f(e^{i\theta})$ is a hyperfunction on ∂D if and only if $f(e^{i\theta})=\sum_{n=-\infty}^{\infty}c_n e^{in\theta}$ where $\limsup_{|n|\to\infty} \sqrt[|n|]{|c_n|}\leqq 1$ and the series converges, of course, in the sense of hyperfunctions.

If one deforms the unit circle homotopically to a curve Γ, both Sato's and