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Besov spaces and analytic semigroups of linear operators

Dedicated to Professor Hiroshi Fujita on his 60th birthday

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Introduction and main results.

It is known that for a strongly continuous function f on an interval $I=(a, b)$ with values in a Banach space X and an analytic semigroup e^{-tA} of linear operators in X the function

$$F(t) = \int_a^t e^{-(t-s)A} f(s)ds \tag{1}$$

is not strongly differentiable in I except that X has a special property or A is bounded in X (see Baillon [**1**]). To guarantee strong differentiability of F we have to assume more smoothness condition on f than strong continuity. Crandall-Pazy [**2**] proved that F is strongly differentiable in I if

$$\int_0^\delta \sup\{\|f(t)-f(s)\|_X ;\ \ t,\ s\in K,\ |t-s|\leqq h\}\frac{dh}{h} < \infty \tag{2}$$

for any compact interval K contained in I. In particular, if f is Hölder continuous, then F is strongly differentiable. The aim of this note is to give an improvement on this result, that is,

THEOREM A. *Let X be a Banach space, e^{-tA}, $t\geqq 0$, an analytic semigroup of linear operators in X, $I=(a, b)$ a finite open interval $1\leqq p\leqq\infty$, $1\leqq q\leqq\infty$, and let σ be a real number. Assume that f belongs to $B^{\sigma}_{p,q}(I; X)_{\mathrm{loc}}\cap L_1(I; X)$. Then the function F defined by* (1) *belongs to $B^{\sigma+1}_{p,q}(I; X)_{\mathrm{loc}}$.*

Here, for a function space $\mathcal{F}(I; X)$ we denote by $\mathcal{F}(I; X)_{\mathrm{loc}}$ the space of functions f which have property that $\phi f\in\mathcal{F}(I; X)$ for any $\phi\in C_0^\infty(I)$, and by $B^{\sigma}_{p,q}$ we denote Besov spaces (Lipschitz spaces) whose definition will be given in § 1.

To treat the inhomogeneous equation

$$\frac{du}{dt}(t)+Au(t) = f(t), \qquad a<t<b, \tag{3}$$

$$u(a) = x, \tag{4}$$