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A decision method for a set of first order classical formulas and its applications to decision problems for non-classical propositional logics

Dedicated to Professor Shôji Maehara for his sixtieth birthday

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I. Main theorem.

Let L be the first order classical predicate logic without equality. We assume that L has a fixed binary predicate symbol R, unary predicate symbols $P_1, \cdots, P_N$ and no other non-logical constant symbols. R-free formulas are formulas in L which has no occurrences of R. R-positive formulas are formulas in L which has no negative occurrences of R. R-formulas are formulas defined inductively as follows:

(1) All R-free formulas are R-formulas;

(2) If A and B are R-formulas, then $\neg A$, $A \wedge B$, $A \vee B$, $A \supset B$ are all R-formulas;

(3) If $A(x)$ is an R-formula and x is a free variable not occurring in $A(v)$, then $\forall v A(v)$, $\forall v(R(x, v) \supset A(v))$, $\forall v(R(v, x) \supset A(v))$, $\exists v A(v)$, $\exists v(R(x, v) \wedge A(v))$, $\exists v(R(v, x) \wedge A(v))$ are all R-formulas.

By R-quantifiers, we denote the quantifiers of the form:

$$\forall v(R(x, v) \supset \cdots v \cdots), \qquad \forall v(R(v, x) \supset \cdots v \cdots),$$
$$\exists v(R(x, v) \wedge \cdots v \cdots), \qquad \exists v(R(v, x) \wedge \cdots v \cdots),$$

where $\cdots v \cdots$ has no occurrences of the free variable x. Then, R-formulas are formulas obtained from R-free formulas by applying propositional connectives, quantifiers and R-quantifiers.

For each R-formula A, let $R\text{-deg}(A)$ be the non-negative integer, called the R-degree of A, defined as follows:

(1) $R\text{-deg}(A) = 0$ if A is R-free.

(2) $R\text{-deg}(\neg A) = R\text{-deg}(A)$,

$R\text{-deg}(A \wedge B) = R\text{-deg}(A \vee B) = R\text{-deg}(A \supset B) = \max\{R\text{-deg}(A), R\text{-deg}(B)\}$,

(3) $R\text{-deg}(\forall v A(v)) = R\text{-deg}(\exists v A(v)) = R\text{-deg}(A(x))$, and