J. Math. Soc. Japan
Vol. 42, No. 1, 1990

An invariant of manifold pairs and its applications

By Mikiya MASUDA

(Received May 23, 1988)
(Revised Sept. 14, 1988)

§ 0. Introduction.

Following [3], [6] let $\Theta^{m,n}$ be the set of h-cobordism classes of pairs (S^m, K) consisting of an oriented homotopy n-sphere K embedded in the oriented m-sphere S^m. It forms an abelian group under connected sum of pairs and the inverse element of (S^m, K), denoted by $-(S^m, K)$, is given by reversing both orientations of S^m and K. In case $m-n\geqq 3$ and $n\geqq 5$, $\Theta^{m,n}$ can be regarded as the isotopy classes of such pairs (S^m, K) by the h-cobordism theorem for pairs. Henceforth we will assume $m-n\geqq 3$ and $n\geqq 5$.

The group $\Theta^{m,n}$ is well understood by the work of J. Levine [6]. A result of [6] says that $\Theta^{m,n}$ has a free part of rank one if and only if $n+1\equiv 0 \pmod 4$ and $3(n+1)\geqq 2m$, and is finite otherwise. Moreover Levine's work implicitly says that in case $3n\geqq 2m$, there is a homomorphism called the signature of knots

$$\sigma : \Theta^{m,n} \longrightarrow \boldsymbol{Q}$$

and that

(0.1) the kernel of σ is finite.

When there is a Seifert surface for K, $\sigma(S^m, K)$ is defined as the signature of the Seifert surface. It is easily checked that the value is independent of the choice of a Seifert surface (here we need the assumption $3n\geqq 2m$). Moreover it immediately follows from the definition that the signature of a Seifert surface is additive with respect to connected sum of pairs. Every knot does not have a Seifert surface, but certain times connected sum of it necessarily has a Seifert surface. Hence one can extend the domain of σ to the whole group $\Theta^{m,n}$ by virtue of the additivity property of signature with respect to connected sum.

In this paper we intend to extend the domain of σ to a more general family of pairs (M, F) consisting of a connected, closed, oriented m-dimensional smooth manifold M and a connected closed oriented n-dimensional smooth submanifold F of M. We require this additivity property:

(AP) $$l((M_1, F_1)\#(M_2, F_2)) = l(M_1, F_1)+l(M_2, F_2).$$