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On toroidal groups

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§ 0. Introduction.

A toroidal group is a quotient $X=\boldsymbol{C}^n/\Gamma$ of $\boldsymbol{C}^n$ by a lattice Γ such that X has no non-constant holomorphic function ([**11**]). Morimoto considered a connected complex Lie group without non-constant holomorphic functions and called it an (H, C)-group ([**12**]). Since every (H, C)-group is commutative, the (H, C)-groups are exactly the toroidal groups.

It is a well known result that for a complex torus $\boldsymbol{T}$ the following are equivalent:

(1) $\boldsymbol{T}$ is an abelian variety.

(2) $\boldsymbol{T}$ has a positive line bundle.

(3) $\boldsymbol{T}$ is projective algebraic.

In the previous paper [**1**] we obtained a similar result for a toroidal group X under the condition $\dim H^1(X, \mathcal{O}_X)<\infty$.

One of the purpose of this paper is to drop the above condition (see Theorem 4.6). This contains answers to problems of the structure and of the global embedding of weakly 1-complete manifolds in the case of toroidal groups (see [**1**]). Another is to prove the meromorphic reduction theorem for toroidal groups (Theorem 5.1). As a by-product we obtain that for a topologically trivial holomorphic line bundle L over a toroidal group $X=\boldsymbol{C}^n/\Gamma$, $H^0(X, \mathcal{O}(L))\neq\{0\}$ if and only if L is analytically trivial (Corollary 3.3). By different methods Huckleberry and Margulis [7] proved it.

§ 1. Preliminaries.

A discrete subgroup Γ of $\boldsymbol{C}^n$ is called a lattice in $\boldsymbol{C}^n$. Let $p_1=(p_{11}, p_{21}, \cdots, p_{n1}), \cdots, p_r=(p_{1r}, p_{2r}, \cdots, p_{nr})\in\boldsymbol{C}^n$ be generators of Γ, where $r=\operatorname{rank}\Gamma$. An (n, r)-matrix

$$P=({}^tp_1\ {}^tp_2\ \cdots\ {}^tp_r)$$

is called a period matrix of Γ, or also of $X=\boldsymbol{C}^n/\Gamma$. Two period matrices P and P' are said to be equivalent if there exist $A\in GL(n, \boldsymbol{C})$ and $M\in GL(r, \boldsymbol{Z})$