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On Jacobian fibrations on the Kummer surfaces of the product of non-isogenous elliptic curves

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Introduction.

Let X be a Kummer surface obtained by the minimal resolution of the quotient surface of the product abelian surface $E\times F$ by the inversion automorphism, where E and F are arbitrarily fixed complex elliptic curves which are *not mutually isogenous*. As is well-known, X is an algebraic $K3$ surface.

This paper is concerned with Jacobian fiber space structures on X, i.e., elliptic fiber space structures with a section on X, or in other words, structures as an elliptic curve over $\boldsymbol{C}(\boldsymbol{P}^1)$. By $\mathcal{J}_X$ we denote the set of all Jacobian fibrations of X.

Let us recall that any elliptic fibration of X is given by the morphism $\Phi_{|\Theta|}: X\to\boldsymbol{P}^1$ defined by the complete linear system $|\Theta|$ which contains a divisor having the same type as a non-multiple singular fiber of an elliptic surface. By definition, an irreducible curve C is a section of $\Phi_{|\Theta|}$ if and only if C satisfies $C\cdot\Theta=1$. We note that every section of $\Phi_{|\Theta|}$ is a nodal curve, i.e., a non-singular rational curve whose self-intersection number is -2. The group $\mathrm{Aut}(X)$ acts on $\mathcal{J}_X$ in an obvious manner; $f: \Phi_{|\Theta|}\to\Phi_{|f(\Theta)|}$ for $f\in\mathrm{Aut}(X)$.

By Sterk [**12**], the orbit space $\mathcal{J}_X/\mathrm{Aut}(X)$ is finite, i.e., the number of non-isomorphic Jacobian fibrations of X is finite.

The purpose of this paper is to describe all Jacobian fibrations of X modulo isomorphism, or saying more clearly, to find a minimal complete set of representatives of the orbit space $\mathcal{J}_X/\mathrm{Aut}(X)$.

As a first consequence of this paper, we see that $\mathcal{J}_X$ is divided into eleven $\mathrm{Aut}(X)$-stable subsets $\mathcal{J}_1, \cdots, \mathcal{J}_{11}$ by types of the singular fibers, and the Mordell-Weil group of its member is calculated for each $\mathcal{J}_m (m=1, \cdots, 11)$ as follows (Table A, Theorem (2.1) in §2). Here, for example, by $2\mathrm{I}_8+8\mathrm{I}_1$ we mean two singular fibers of type I_8 (Kodaira's notation) and eight singular fibers of type I_1.

We note that there exist infinitely many nodal curves on X since X has a Jacobian fibration whose Mordell-Weil group is an infinite group by Table A. From this fact we can construct *infinitely many* Jacobian fibrations of X.