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Estimates on the stability of minimal surfaces and harmonic maps

By Makoto SAKAKI

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0. Introduction.

Let $f: M \to N$ be a minimal immersion of a manifold M into a Riemannian manifold N. We say that M is stable if the second variation of the volume functional of M is positive for every variation of f which leaves the boundary values fixed. Let $N^n(a)$ denote the n-dimensional simply-connected space form of constant curvature a. Barbosa and do Carmo [**1**], [**2**] discussed the stability of simply-connected compact minimal surfaces with piecewise C^1 boundary in $N^n(a)$, whose result was improved in our previous paper [**9**] as follows.

THEOREM 0.1 ([**9**]). *Let $f: M \to N^n(a)$ be a minimal immersion of a 2-dimensional simply-connected compact manifold M with piecewise C^1 boundary into $N^n(a)$. If the second fundamental form A of f satisfies $\int_M (|a|+(1/2)|A|^2)dM < (4/3)\pi$, then M is stable.*

When $a \geqq 0$, Theorem 0.1 is proved in a little different way (cf. [**1**], Hoffman and Osserman [**6**]). In [**2**] it is asked if the argument of Theorem 0.1 can be generalized or not for a general ambient space. The first aim of this paper is to give a positive answer to this question. Let G_2N denote the Grassmann bundle over a Riemannian manifold N of 2-dimensional tangent subspaces to N. The Riemannian structure of G_2N is defined in Section 1.

THEOREM 0.2. *Let $f: M \to N$ be a minimal immersion of a 2-dimensional simply-connected compact manifold M with piecewise C^1 boundary ∂M into a Riemannian manifold N. Suppose that the sectional curvature of N is bounded and the sectional curvature of G_2N is bounded from above. Then there is a positive constant c_1 depending only on N such that if the second fundamental form A of f satisfies $\int_M (1+(1/2)|A|^2)dM < c_1$, then M is stable.*

If we omit the hypothesis that M is simply-connected and assume the positivity of the injectivity radius of G_2N, we obtain the following estimate (cf.