J. Math. Soc. Japan
Vol. 41, No. 3, 1989

Two results in the affine hypersurface theory

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(Received April 4, 1988)
(Revised June 10, 1988)

Introduction.

In this paper, we shall show two results in the affine geometry.

The first theorem is on the structures of affine spheres in 3-dimensional real vector space $\boldsymbol{R}^3$. The complete affine spheres have been already classified by Calabi [2], Pogorelov [8], Cheng and Yau [4] and Sasaki [9]. On the other hand, we know little on the structures of affine spheres without the condition of completeness. We study non-complete affine spheres which satisfy a certain curvature condition for their affine metric.

In the second theorem, we shall study the hypersurfaces obtained as the graphs of the affine normal vector fields of affine hypersurfaces. It will be shown that the hypersurfaces thus obtained satisfy an integral formula in terms of the affine invariants. Conversely, it will be proved the integral formula is also the sufficient condition for affine hypersurfaces to be constructed in this way.

To explain our results more precisely, we review here some notations and facts in the affine geometry. (For more details, see [1], [2] and [5].) Let x: $M \rightarrow \boldsymbol{R}^{n+1}$ be a strictly convex hypersurface of $\boldsymbol{R}^{n+1}$. If we choose a vector field ξ of $\boldsymbol{R}^{n+1}$ along M such that $T(\boldsymbol{R}^{n+1})|M=T(M)+\boldsymbol{R}\cdot\xi$, we can define the induced affine connection ∇ and the second fundamental form h as follows: for arbitrary vector fields X and Y on M,

$$D_X Y=\nabla_X Y+h(X, Y)\cdot\xi,$$

where D is the standard affine connection of $\boldsymbol{R}^{n+1}$, and the vector field $\nabla_X Y$ is tangent to M.

Because of the strict convexity of M, h is definite and we can determine ξ uniquely by the following two conditions:

(i) For any vector field X of M, the vector field $A(X)=-\nabla_X\xi$ is tangent to M.

(ii) Let vol be the standard volume element of $\boldsymbol{R}^{n+1}$, and $(e_1, \cdots, e_n)$ the frame field of M. Then