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Closed orbits of non-singular Morse-Smale flows on S^3

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The closed orbits of a non-singular Morse-Smale flow ([8], p. 798) on S^3 form an indexed link, that is, a link with the index 0, 1 or 2 attached to each component. Although closed orbits are naturally oriented, we do not consider oriented links since the orientation of a closed orbit of a non-singular Morse-Smale flow can be easily reversed by modifying the flow near the closed orbit.

In this paper, we characterize the set of indexed links which arise as the closed orbits of a non-singular Morse-Smale flow on S^3 in terms of a generator and six operations. The generator is the Hopf link with indices 0 and 2 attached to the components, and the operations are, roughly speaking, split sum, connected sum, and cabling.

Since the author first obtained the result, several papers dealing with the topic have appeared ([6], [7], [9]). Of these, the works of Sasano [7] and Yano [9] were independently done, and are contained in the results in this paper.

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§1. Results.

The Hopf link with indices 0 and 2 attached to the components is called the (0, 2)-Hopf link. We prove the following:

THEOREM. *Every indexed link which consists of all the closed orbits of a non-singular Morse-Smale flow on S^3 is obtained from* (0, 2)*-Hopf links by applying the following six operations. Conversely, every indexed link obtained from* (0, 2)*-Hopf links by applying the operations is the set of all the closed orbits of some non-singular Morse-Smale flow on S^3.*

OPERATIONS. For given indexed links l_1 and l_2, we define six operations as follows. We denote by $l_1 \cdot l_2$ the split sum of l_1 and l_2, and by $N(k, M)$ the regular neighborhood of k in M. For other terminologies of knot theory, refer to [5].

I. To make $l_1 \cdot l_2 \cdot u$, where u is an unknot with index 1.
II. To make $l_1 \cdot (l_2 \setminus k_2) \cdot u$, where k_2 is a component of l_2 of index 0 or 2.
III. To make $(l_1 \setminus k_1) \cdot (l_2 \setminus k_2) \cdot u$, where k_1 is a component of l_1 of index 0,