

Expansive homeomorphisms with the pseudo-orbit tracing property of n -tori

Dedicated to Professor Yukihiro Kodama on his 60th birthday

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§1. Introduction.

The strongest useful equivalence for the study of orbit structures of homeomorphisms will be topological conjugacy. Our investigation will be within the context of the conjugacy problem for homeomorphisms with expansiveness and the pseudo-orbit tracing property (abbrev. POTP). The author proved in [12] that every compact surface which admits such homeomorphisms is the 2-torus and moreover that such a homeomorphism of the 2-torus is topologically conjugate to a hyperbolic toral automorphism. Thus it seems that orbit structures of homeomorphisms of the n -torus will be determined under the assumption of expansiveness and POTP. And so it will be natural to ask whether every homeomorphism with expansiveness and POTP of the n -torus is topologically conjugate to a hyperbolic toral automorphism. An answer of this problem is given as follows.

THEOREM. *Let $f: T^n \rightarrow T^n$ be a homeomorphism of the n -torus. If f is expansive and has POTP, then f is topologically conjugate to a hyperbolic toral automorphism.*

The notion of “ c -map” is introduced for (self-) covering maps. The class of covering maps with this notion is wider than that of homeomorphisms having expansiveness and POTP. Recently in [2] N. Aoki and the author obtain some interesting results for c -maps, which relate to our theorem.

Let (X, d) be a metric space and $f: X \rightarrow X$ be a (self-) homeomorphism. We say that f is *expansive* if there is $c > 0$ (called an *expansive constant*) such that if $x, y \in X$ and $x \neq y$ then $d(f^n(x), f^n(y)) > c$ for some $n \in \mathbf{Z}$. A sequence $\{x_i\}_{i \in \mathbf{Z}}$ of X is a δ -*pseudo-orbit* of f if $d(f(x_i), x_{i+1}) < \delta$ for all $i \in \mathbf{Z}$. A point $x \in X$ ε -*traces* a sequence $\{x_i\}_{i \in \mathbf{Z}}$ of X if $d(f^i(x), x_i) < \varepsilon$ for all $i \in \mathbf{Z}$. We say that f has *POTP* if for $\varepsilon > 0$ there is $\delta > 0$ such that every δ -pseudo-orbit of f is ε -traced by some point of X . Note that if X is compact, then expansiveness