J. Math. Soc. Japan
Vol. 41, No. 2, 1989

A note on fundamental dimensions of Whitney continua of graphs

By Hisao KATO

(Received Sept. 28, 1987)

1. Introduction.

By a *continuum*, we mean a compact connected metric space. Let X be a continuum with metric d. By the *hyperspace* of X, we mean

$$C(X) = \{A \mid A \text{ is a nonempty subcontinuum of } X\}$$

with the *Hausdorff metric* d_H, i.e., $d_H(A, B)=\inf\{\varepsilon>0 \mid U(A, \varepsilon)\supset B \text{ and } U(B, \varepsilon)\supset A\}$, where $U(A, \varepsilon)=\{x\in X \mid d(x, A)<\varepsilon\}$. In [**22**], Whitney showed that for any continuum X there exists a map $\omega: C(X)\to[0, \omega(X)]$ satisfying

(1) $\omega(\{x\})=0$ for every $x\in X$, and

(2) if $A, B\in C(X)$, $A\subset B$ and $A\neq B$, then $\omega(A)<\omega(B)$.

Any such map ω is called a *Whitney map*. We may think of the map ω as measuring the size of a continuum. It is well-known that every Whitney map ω is monotone, i.e., $\omega^{-1}(t)$ is a continuum for each $0<t<\omega(X)$. The continuum $\omega^{-1}(t)$ $(0\leqq t<\omega(X))$ is called a *Whitney continuum*. Note that $\omega^{-1}(0)$ is homeomorphic to X and $\omega^{-1}(\omega(X))=\{X\}$. Naturally, we are interested in the structure of $\omega^{-1}(t)$ $(0<t<\omega(X))$. Let X be a continuum. Then the *fundamental dimension* $\mathrm{Fd}(X)$ of X is defined as follows (see [**1**] or [**16**]): $\mathrm{Fd}(X)=\min\{\dim Z \mid Z$ is a continuum such that Z has the same shape as $X\}$. In particular, if P is a compact connected polyhedron, then $\mathrm{Fd}\, P=\min\{\dim Z \mid Z$ is a compact connected polyhedron such that Z has the same homotopy type as $P\}$.

In [**11**] and [**2**], Kelley and Duda investigated the dimension of $C(G)$ for a graph G. In particular, Duda described and analyzed polyhedral models for hyperspaces of graphs (see [**2**] and [**3**]). In [**5**, (2.4)], we showed that $\omega^{-1}(t)$ is a polyhedron for any graph G, any Whitney map ω for $C(G)$ and $t\in[0, \omega(G)]$ (cf. [**15**]).

In [**5**, (2.9)], we defined an index $n(G)$ for a graph G and showed that if ω is any Whitney map for $C(G)$, then $\mathrm{Fd}\,\omega^{-1}(t)\leqq n(G)-1$ for each t. Also, we showed that Whitney continua of graphs admit all homotopy types of compact connected ANR's ([**7**]).